VON KARMAN CENTER

ADVANCED POWER SYSTEMS DEPARTMENT

MISSION ANALYSIS FOR SOLAR-HEATED HYDROGEN PROPULSION SYSTEM

VOL. II – COMPARATIVE MISSION ANALYSIS

A REPORT TO

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

CONTRACT NO. NAS 7-230

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A COMPARATIVE MISSION ANALYSIS FOR
SOLAR HEATED HYDROGEN PROPULSION SYSTEM

Volume II: Mission Analysis

A REPORT TO

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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AEROJET-GENERAL CORPORATION
A SUBSIDIARY OF THE GENERAL TIRE & RUBBER COMPANY

CONTRACT FULFILLMENT STATEMENT

This final report was prepared by the Advanced Power Systems Department of the SNAP-8 Division, Aerojet-General Corporation, and is submitted in fulfillment of National Aeronautics and Space Administration Contract No. NAS 7-230.

ABSTRACT

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This two-volume final report was prepared in compliance with a National Aeronautics and Space Administration Contract NAS 7-230, "A Comparative Missions Analysis for Solar Heated Hydrogen Propulsion System."

Volume I, "System Weight Analysis," presents an analytic tool and computational procedure for evaluating three functionally equivalent solar hydrogen propulsion systems. Specific numerical examples are presented and comparisons are made. The combination of components that produces the lowest system weight is defined. The basic vehicle-SHPS integration problems are discussed and a model that later becomes the basis for all computations is selected.

System weights, in terms usable for mission analysis, are prepared for Advanced Chemical and Nuclear-Heated Hydrogen Systems. Weights of the Nuclear Electric Propulsion System and the Solar Electric Propulsion System are also presented.

Volume II, "Mission Analysis," presents the energy requirements for the space maneuvers and also presents a method of comparing the various propulsion systems of interest.

The operations and maneuvers for which data are presented are: Author

Orbital Operations

- Altitude Change
- Eccentricity Change
- Plane Change
- Position or Epoch Change
- Station Keeping
- Attitude Control

Interplanetary Operations

- Transfer Between Plants
- Capture
- Co-orbiting
- Fly-by

Achieving Independent Heliocentric Orbits

- Circular Ecliptic Plane
- Out of Ecliptic
- Eccentric Orbit Solar Probe

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Volume II

I. INTRODUCTION

This is Volume II of a two volume report prepared for SNAP-8 Division Aerojet-General Corporation by Space General Corporation.

A. OBJECTIVES AND GUIDELINES

The principal objective of this study has been to describe the performance of solar hydrogen propulsion systems and competitive systems, principally chemical and nuclear hydrogen, in terms of payload and time, over the range of practical space maneuvers. From such performance data, practical space missions for the SHPS can be synthesized, and the range of parameters within which SHPS could operate most competitively can be defined.

To fulfill the above-stated objectives, three basic tasks were necessary:

(1) System analysis to determine weight dependencies of the various propulsion system types.

(2) Mission analysis to determine the energy requirements of all space maneuvers of interest.

(3) Analysis of comparison techniques to determine a practical procedure for propulsion system comparison.

B. ORGANIZATION OF VOLUME II

This report, Volume II, is organized in the following manner.

First, an account is presented of the work tasks performed and the significance of those tasks to the over-all objective.

Second, an analysis of results is presented to show how the SHPS compares with other systems. An effort has been made to point out the general ranges of parameters which are most favorable to SHPS, and the specific missions to which these parameter values correspond.

Third, a description is given of the areas in which the methods of analysis should be refined, and additional data (mission and system) should be generated.

The actual compilation of study data and the methods of utilizing and interpreting it are included in the Appendix.

The Appendix contains the three major categories referred to previously:

(1) Procedure for Synthesis of Missions
(2) Mission Requirements
(3) System Weight and Performance Data

The mission data are divided according to central body, such as Moon, Earth, Mars, etc. The heliocentric maneuver data are included in the section designated "Sun." All synthesis of interplanetary missions is included in the main text. The system weight data necessary for mission analysis have been summarized from Volume I and are included in Part III of the Appendix.

II. SUMMARY OF TASKS ACCOMPLISHED

Table II-1 is a list of topics treated in the course of the mission study, which meets the requirements of the work statement. The listed items are primarily types of missions or maneuvers, and phenomena affecting vehicle performance in these maneuvers. The actual tasks accomplished in regard to these technical areas are described as follows.

A. PROCEDURES FOR EVALUATING PROPULSION SYSTEMS

The principal problem involved in the mission analysis was that of providing a reasonable, easy-to-use, measuring stick for evaluating propulsion system performance in a given task and for easily comparing two systems. Since a mission task can be described in alternate ways, using different variables, a procedure was required which was flexible enough to handle a problem in a number of different ways. Several methods were developed for this purpose, but the most convenient of these was developed in the final quarter of the study. It consists of three graphs.

1. ΔV as a function of mission end conditions and time requirement.

2. Plot of the relationship between I_{sp}, ΔV, W_p/W_o, η and W_μ/W_o (defined in Figure II-1).

3. Graph of the relationship between thrust level, propellant fraction, and propellant weight.

TABLE II-1

TOPIC INDEX

Figure II-1 is a combination of these graphs specifically suited to the analysis of SHPS characteristics in connection with probes from earth orbit to other planets and the moon. The plot on the left of the figure presents the ΔV and time requirements of the various probe missions. The plot in the lower right corner presents system characteristics, and in the center is a nomograph for relating mission requirements to system characteristics and payload. The plot on the left may be replaced with any mission ΔV plot in the Appendix, and the plot on the lower right may be replaced with the propellant fraction vs propellant weight plot of an alternate system configuration or another type of propulsion system. Figure II-1 is presented primarily to show the principal factors of a mission synthesis in an uncomplicated manner. The composite shows how certain parameters become constrained when the values of certain others are selected. The relationship of the variables is shown in the diagram of Figure II-2. The establishment of any two variables in a straight line establishes the third.

From Figure II-1 an important trade-off can be observed between ΔV and η. The higher the F/W ratio, the lower the ΔV requirement. The establishment of these two (for a system of known I_{sp}) fixes the η available. If η could be picked independently, it would be chosen as high as possible to get maximum benefit from the propellant, but this would fix the ΔV and thus the F/W at a less-than-optimum value. The dilemma could be resolved by simply plotting payload ratio as a function of F_o/W_o ratio for the I_{sp} involved, and selecting the F_o/W_o that produces the highest payload. Such a plot would, however, not show the practical characteristics, propellant fraction and propellant ratio, or the thrust-propellant weight relationship.

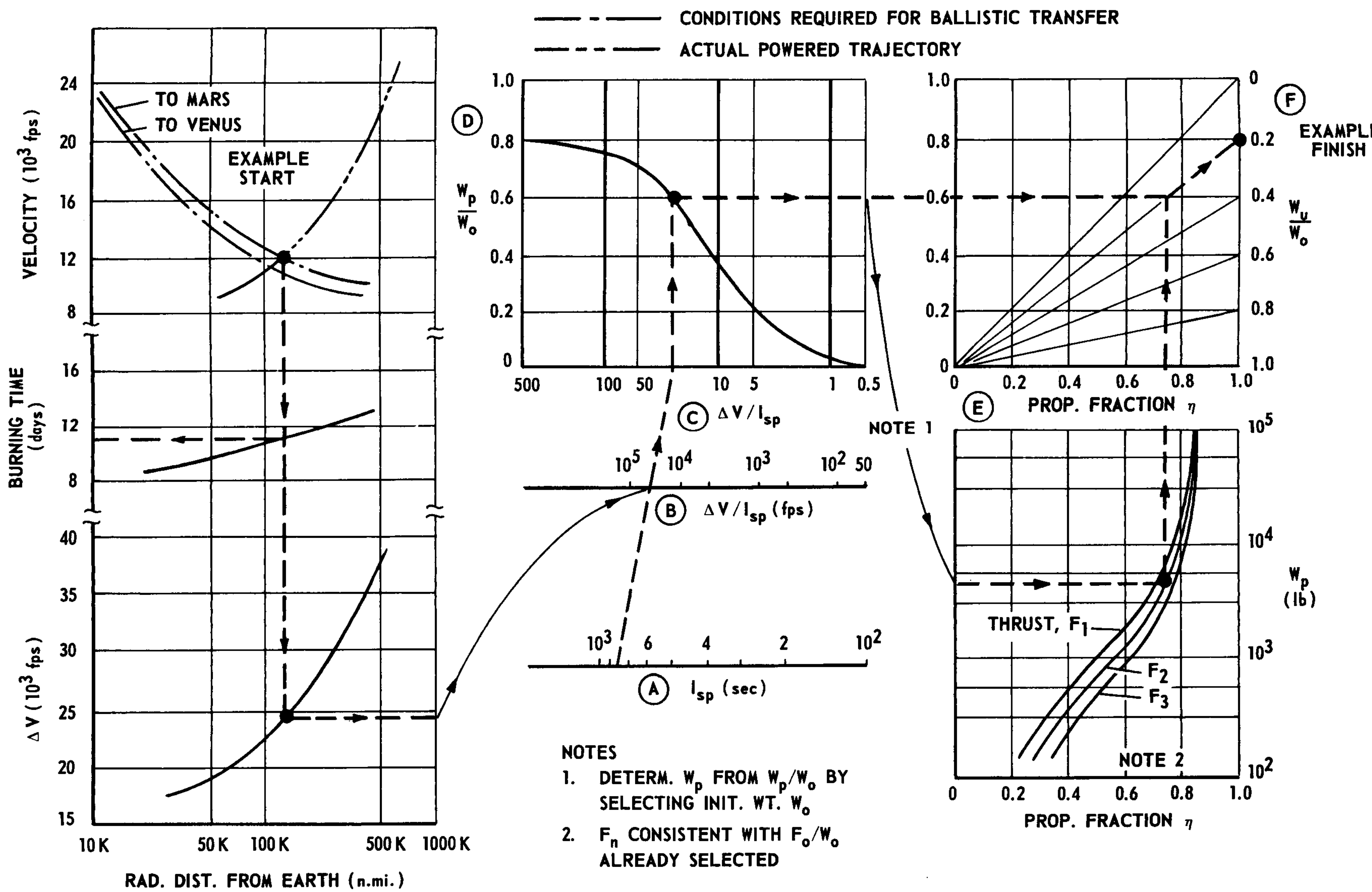


FIGURE II-1. Graphical Determination of Payload Ratio

ESTABLISHMENT OF VALUES OF ANY TWO VARIABLES IN A STRAIGHT LINE FIXES THE VALUE OF THE THIRD

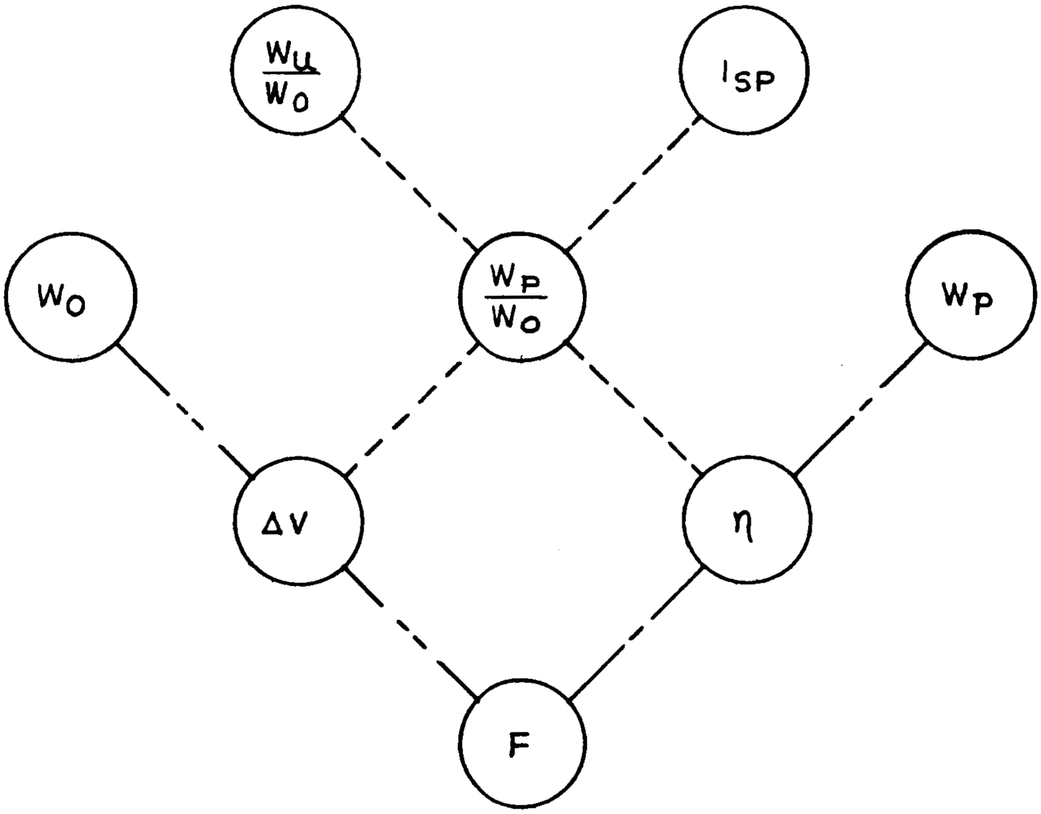


Figure II-2. Diagram of Variables in Mission Analysis

Also, an expression of W_μ/W_o in terms of F_o/W_o could be evaluated for maximum W_μ/W_o by setting the derivative equal to zero.

The fact that the payload ratio can be expressed in terms of F_o/W_o (for a given mission and I_{sp}) is verified in the following equations:

$$\frac{W_\mu}{W_o} = 1 - \frac{1}{\eta}\left(1 - e^{\frac{\Delta V}{gI_{sp}}}\right) \tag{1}$$

$$\eta = \frac{W_P}{W_{tanks} + W_P + W_{thrust}} = \frac{1}{\frac{W_{tanks}}{W_P} + 1 + \frac{W_{thrust}}{W_P}} \tag{2}$$

$$W_{tanks} = K_3 W_P^{\,y} \tag{3}$$

$$W_{thrust} = K_3 F_o \tag{4}$$

$$\eta = \frac{1}{K_3 W_P^{\,y-1} + 1 + K_4\, F_o/W_P} \tag{5}$$

$$\frac{F_o}{W_P} = \frac{F_o/W_o}{1 - e^{-\Delta V/gI_{sp}}} \tag{6}$$

$$\Delta V = K_1 \left(\frac{F_o}{W_o}\right)^X + K_2 \qquad \text{(empirical)} \tag{7}$$

Insertion of (5), (6), and (7) in (1) results in W_μ/W_o as a function of F_o/W_o for $y \approx 1$.

The estimated value of y for the SHPS tanks is 1.082.

The value of F_o/W_o that yields maximum payload ratio for the specific case represented in Figure II-1 can be determined rather quickly by trial and error selection of F_o/W_o in that figure.

The procedure for using the nomograph portion of Figure II-1 is described in detail in the procedural section of the Appendix.

B. MISSION ΔV REQUIREMENTS

A substantial part of the work effort in the SHPS study was devoted to the development of mission ΔV data. Analytical closed form expressions were used to provide ΔV for cases in which orbit eccentricity changes a negligible amount during a maneuver (very low-thrust acceleration) and for high-thrust cases which approximate instantaneous velocity correction. For other cases (chiefly characteristic of the SHPS), a numerical integration computer program was used. Since the SHPS follows a path through space that is different from that of either high-thrust systems or electrically propelled vehicles, the amount of energy required for a given mission may be expected to be different also. To obtain a realistic view of the SHPS vehicle's trajectory, a special trajectory program was designed. The program has two basic forms:

(1) Heliocentric circumferential thrusting with constant thrust magnitude

(2) Planetocentric tangential thrusting with constant thrust level

In the program, orbital parameters are calculated at every increment of time during thrusting, and computing stops when the desired apsidal radius is obtained, as in the heliocentric case, or when the required posiiton-velocity condition is reached, as in the planetocentric case. Mass change due to propellant consumption, and thrust magnitude change due to radial motion with respect to the sun, are both accounted for in the trajectory program. The ΔV, or integral of the thrust acceleration, is continually recorded in the program, to facilitate comparison. The program is based on the two-dimensional, restricted two-body problem. Data were obtained from this program for hyperbolic escape from the moon and all planets out to, and including, Mars. In addition, data for heliocentric transfer inward and outward from the Earth's orbit were obtained.

The hyperbolic escape trajectories were computed on the basis of tangential thrust, and the heliocentric transfers, on the basis of circumferential thrust. The latter was considered important as the extreme in simplicity for a SHPS maneuver. Tangential thrusting necessitates freedom of rotation between the thrusting nozzle and the focal axis of the solar concentrator, which must be continually oriented toward the sun. The parabolic escape trajectories from the earth were made for three different thrust-to-weight ratios, 10^{-3}, t x 10^{-4}, and 2.5 x 10^{-4}, while those from other planets were made only with a ratio of 10^{-3}. The escape trajectory runs were complicated by the existence of oscillations in the trajectory which, although a natural occurrence in low-thrust mechanics, demand special consideration regarding initial conditions and selection of computing interval. An initial orbit altitude of 300 nm was assumed for all computer-calculated escapes. A wide range of initial altitudes is represented in the data obtained by closed form solution.

C. SYNTHESIS OF INTERPLANETARY MISSIONS

Interplanetary missions were synthesized in the following manner. For high-thrust cases, the ΔV for achieving the correct hyperbolic excess velocity for a heliocentric ellipse from Earth to another planet was computed by means of the conventional equation for instantaneous correction. This is the probe requirement. To this ΔV was added the ΔV for high-thrust hyperbolic escape from the planet to earth. The total is the capture requirement. The co-orbiting requirement was obtained by adding the requirement for escape from the earth to the hyperbolic excess velocity (difference between the orbital velocity of the destination planet, and the heliocentric velocity of the transfer ellipse at the planet) at the destination planet.

For low-thrust SHPS missions, plots were made of the velocity required at each altitude, above the Earth, for the vehicle to be on a correct hyperbolic escape trajectory to the destination planet. Against the same altitude scale was plotted an actual velocity history of the vehicle taken from the computer program. The intersection of the two curves represents the achievement of the required velocity-distance combination and would be the appropriate point to cease thrusting. The ΔV corresponding to this point was adopted as the probe ΔV for SHPS. As in the high-thrust case, the ΔV for hyperbolic escape by low thrust from the destination planet to earth was added to the probe ΔV to obtain the total ΔV for capture. For low-thrust co-orbiting missions the hyperbolic excess at the destination planet was added to the probe ΔV.

A more accurate method would consist of running heliocentric trajectories outward from the inner planets and inward from Mars, and adding those ΔV's to the probe ΔV to obtain co-orbit requirements. Funding limitation prohibited this approach. However, it is shown later (Tables III-9 and III-10 in section III, "Results of Analysis") that the error involved is very small.

The use of the two-body planet-centered hyperbolic escape trajectory is valid only if the required escape conditions are reached while the vehicle is reasonably close to the planet. A commonly used measure of the limit of validity of the two-body unperturbed model is the sphere of influence, which was used in this study.

Hyperbolic escape to all planets was achieved within the sphere of influence of the Earth, using 10^{-3} thrust acceleration. For Mercury probes using thrust acceleration less than 10^{-3} g, the ΔV may be approximated by adding the ΔV for parabolic escape from Earth to the ΔV requirement for transfer from Earth escape conditions to heliocentric ellipse with perihelion at the radius of Mercury.

D. AUXILIARY OPERATIONS

A complete set of parametric ΔV data was obtained for the tasks of attitude control and station keeping. These data are presented in Section II of the Appendix. The attitude control graphs provide thrust and total impulse data, as well as equivalent ΔV (Appendix). The station keeping graph is essentially a plot of integrated perturbations versus altitude, orbit inclination, and vehicle dimensions.

III. RESULTS OF ANALYSIS

A. SUMMARY

The main points of the results are summarized as follows:

1. The SHPS and the nuclear-hydrogen system are applicable to different vehicle weight ranges. The former can be expected to provide favorable performance in vehicles below approximately 100,000 pounds and the latter can operate favorably only in vehicles above that range. In the low range, the large value of minimum weight of the nuclear system results in a low propellant fraction. In the high ranges, the thrust limitation of the SHPS results in high ΔV and long burning times and trip durations.

2. In the comparison of the cryogenic chemical system and the SHPS, the payload advantage shifts from one system to another over the range of maneuvers. The maneuvers in which there is little ΔV difference between high- and low-thrust methods favor the SHPS. Those in which the ΔV difference is large favor the chemical system.

3. The maneuvers in which the ΔV difference between high-thrust and low-thrust operation is relatively small are orbital altitude changes and heliocentric transfers. The ΔV difference is relatively large (favoring high-thrust operation) in escape maneuvers, principally Earth escape. The comparative performance in any mission depends upon the proportion in which these two types of maneuver exist.

4. The propellant fraction of the SHPS is generally lower than that attainable with chemical systems. Present estimates indicate a limit of about 0.90 for a single stage. Pro-rating the thrusting system

weight over a series of tank stages results in average propellant fractions as high as 0.92.

5. In the appropriate weight range, transit time for SHPS is not appreciably longer than that for high-thrust systems, in interplanetary missions. The mission consists mostly of coasting. Probe missions have burning times in the range of 3 - 15 days with SHPS.

6. The necessary burnout conditions for interplanetary probes can be reached by SHPS usually within one-half million nm from the Earth. Thus the variation in thrust with distance from the sun has little effect on SHPS probe missions, that is, missions in which only one thrusting period is involved at the beginning of the heliocentric ellipse.

B. DETAILED DISCUSSION

The study showed that thrust-to-weight ratios available with SHPS are sufficiently high to facilitate execution of orbital and interplanetary missions in reasonably short times, slightly higher than those associated with high thrust but considerably lower than those generally associated with electric engines. The I_{sp} advantage of the hydrogen system allows the SHPS to produce payloads comparable to, and in some cases higher than, those of LOX Hydrogen (chemical) systems.

The quantity most in question regarding the relative performance of the advanced propulsion systems is the propellant fraction. The SHPS appears to have a maximum propellant fraction of 0.92, due to tankage design limitations, and a minimum weight characteristic of about 87 $\frac{\text{lb weight}}{\text{lb thrust}}$ for the thrusting subsystem. How much these quantities can be improved by development is difficult to predict. Curves of SHPS propellant fraction appearing in this report are based on a specific weight of 127 $\frac{\text{lb}}{\text{lb thrust}}$. The propellant fraction of the nuclear hydrogen system is predicted to be about as high as 0.9, not including shielding. The reduction in propellant fraction due to shielding has not been determined. It is evident that a propellant fraction of 0.9 would render the nuclear system superior to SHPS and chemical systems for any mission, since it enjoys the ΔV advantage characteristic of the high-thrust system and the I_{sp} advantage of the hydrogen system. The smallest nuclear engine, however, will be so large that propellant fractions in the neighborhood of 0.9 can only be obtained with vehicles weighing several hundred thousand pounds. The SHPS is not capable of performing missions in reasonable times with such vehicle weights. An additional factor in comparison is, of course, that the SHPS is realizable at a much earlier date than a nuclear system.

Table III-1 shows the payloads obtainable with SHPS if a 0.9 propellant fraction can be maintained, and with LOX-Hydrogen in interplanetary missions. Table III-2 shows the values (planetocentric) of time and ΔV corresponding to the payload data of Table III-1. Table III-3 contains the time and ΔV for heliocentric phases of operation. Table III-4 contains the ΔV totals for the missions represented in Table III-1. The mission requirements have been greatly simplified for this limited study and can therefore be expected to be greater than shown, but the table provides a good comparison, and an approximate indication of actual payload. As shown in the table, SHPS can have a payload advantage for several of the co-orbit and capture missions. The durations of these missions are approximately the same as for high thrust since the largest contributor to mission duration is the coast period. It was expected that missions to Mercury would be favorable to the SHPS because of the increase in solar flux as the vehicle nears the sun. However, for the probe cases, the coast period begins before the vehicle gets appreciably closer to the sun than 1 au. Likewise, for Mars probes the increased distance from the sun has little effect on the probes. An effect does exist, of course, for the co-orbiting and capture cases, but a detailed analysis of the effect was not made during this study.

The SHPS data of Table III-1 are based upon a thrust-to-weight ratio of 10^{-3}. This represents a practical upper limit for SHPS. Lower ratios, as low as 10^{-4}, are practical from the standpoint of vehicle weight and mission time, although higher ΔV's and therefore smaller payloads result. The increase in ΔV requirement due to lowering the F_o/W_o ratio can be seen in Figure II-1 of the previous section. The burning time increases when the thrust-to-weight ratio is lowered, reducing the predicted reliability somewhat.

For values of F_o/W_o below 10^{-4} the burning time becomes a significantly large part of the total mission time and the total time may become objectionably larger. Data shown in this report for thrust-to-weight ratios below 2.5×10^{-4} were obtained by extrapolation.

Complete data for 0.5×10^{-3} and 2.5×10^{-4} are not yet available for the other planets; therefore, a complete payload table cannot be made for those values. In future study, the data for these and other low thrust-to-weight ratios will be obtained either by additional computer runs or by mathematical analogy. It has been basically assumed that the variation of ΔV with thrust-to-weight ratio will be found to be a smooth curve and that the variation at other planets will be similar to that at Earth. For terminal maneuvers at the inner planets, the thrust-to-weight ratio is obviously larger than that existing at the beginning of the mission; therefore the 10^{-3} thrust-to-weight ratio has been considered a reasonable assumption for roughly determining the ΔV at Venus and Mercury.

In the category of results of this mission analysis project, note that several very useful tools of a general nature have been developed. The chief value of these tools lies in the speed with which information (particularly payload information) can be obtained. A simple plot has been presented for determining payload ratio for given inputs ΔV, I_{sp}, and η. An extensive catalog of mission ΔV's has been provided from which to obtain the ΔV input; and the variation of η with thrust level, propellant weight, and type of system has been presented to facilitate selection of that input.

Certain missions have been omitted from this study, due to their complexity. An example is the optimized planet-to-planet transfer. Such optimization would apply primarily to cases involving low thrust-to-weight ratios, in which thrusting has to be performed over an appreciable portion of the trajectory. There was not sufficient time to determine thrust orientation programs for such missions. It was not possible in the allotted time to study the completed data sufficiently to postulate approximate relationships which would enable rule of thumb estimates of ΔV requirements. Natural perturbations were omitted from interplanetary programs in the interest of getting approximate answers in a minimum time.

Escape trajectories for F_o/W_o less than 2.5×10^{-4} were not run due to time limitations. Oscillations in the escape trajectories necessitated smaller and smaller time increments in the integration, causing large increases in required computer time. The initial 300 nm orbit had to be modified slightly (made slightly eccentric) so that initial conditions could be chosen to minimize the oscillatory effect. The causes and dependencies of the oscillatory behavior were being studied toward the end of the project; but additional study is required to clarify the effect of the oscillation phenomenon on the selection of F_o/W_o ratio.

Data pertaining to several phases and modes of thrusting had to be combined to provide the ΔV requirements necessary to produce payload information such as that of Table III-1. These supporting data were summarized in Tables III-2, III-3 and III-4.

Additional data were generated, immediately before the publication of this report, which present payload and thrusting-system weights lumped together and determined on the basis of staged tanks and a 60,000 lb initial weight. The data are presented in Table III-5. The range of payload weights resulting from the substitution of various weights for the 15-lb (thrust) thrusting system can be easily seen. In each case payload will be reduced by approximately 1305 lb considering an 87 lb/lb specific weight for the thrusting system.

One of the chief causes of difficulty generally encountered in comparing propulsion systems is the fact that there are several variables connected with the determination of payload yield. The smallest number of variables is three. The variables are ΔV, η, and I_{sp}. For systems having the same or similar propellant fraction, η, (such as SHPS and LOS-Hydrogen) a table of equivalence between ΔV and I_{sp} can be constructed with actual payload ratio as a parameter. Assuming $\eta = 0.9$ the following table (Table III-6) evolves for two systems with I_{sp} equal to 800 and 430, respectively.

Thus, for practical ΔV levels the higher I_{sp} system can deliver approximately 90% more ΔV for a given payload ratio. This applies to any payload weight.

Such a system, then, would deliver greater payload than the lower I_{sp} system in all missions for which the ΔV difference does not exceed 90.5% of the lower value. Comparing the ΔV advantage, afforded by the higher I_{sp}, with the difference in ΔV between high- and low-thrust methods of operation provides a quick means of comparing a SHPS with a high-thrust chemical system.

It can be concluded, therefore, that a SHPS with a propellant fraction of 0.9 and any reasonable payload fraction can provide at least

80% more ΔV than a chemical (I_{sp} = 430 sec) system with the same propellant fraction. A single-stage SHPS with such a propellant fraction might have a thrust level of one or two pounds and a propellant weight on the order of 3 to 10,000 lb. For transfers from a 100 nm earth orbit up to 40,000 nm the difference in ΔV is less than 5500 fps or 40.7%.

To show the maximum payload advantage achievable with SHPS, Table III-7 was prepared. Table III-7 shows the payload ratio of both the chemical and the SHPS, on the assumption that the mission ΔV is the same for both systems. For the 70,000 fps case, Table III-7 shows that the SHPS can deliver about 18.6 times as much payload as the chemical system.

It is evident that tasks in which the ΔV is the same for high thrust as it is for low thrust will appear from time to time in the space effort, and that SHPS is well suited for such tasks. Table III-8 shows ΔV requirements for interplanetary probe missions starting in a near earth orbit. The ΔV's for the high-thrust chemical system are considerably lower than those for SHPS. On the other hand, Table III-9 shows the ΔV's for interplanetary probes starting at earth escape. The ΔV difference between high thrust and SHPS is essentially zero. The SHPS ΔV data in Tables III-8 and III-9 were obtained from the SGC trajectory program, and the high-thrust ΔV data were obtained by means of well established analytical techniques.

TABLE III-1

PAYLOAD RATIOS FOR INTERPLANETARY MISSIONS

(Starting in 300 nm Earth Orbit)

Destination	Low Thrust $\frac{F_o}{W_o} = 10^{-3}$ Probe	Co-Orbit	Capture	Number of Stages	High Thrust $\frac{F_o}{W_o} = 10^{-1}$ Probe	Co-Orbit	Capture
Mercury	0.143			1	0.162		
	0.178	0.030	0.019	2	0.193	0.004	0.002
	0.183	0.038	0.027	3	0.199	0.009	0.006
	0.186	0.042	0.030	4	0.202	0.011	0.008
Venus	0.341	0.206	0.006	1	0.364	0.131	0.113
	0.356	0.233	0.042	2	0.378	0.166	0.150
	0.361	0.239	0.110	3	0.382	0.173	0.158
	0.363	0.242	0.131	4	0.384	0.177	0.161
Moon	0.422	0.400	0.339	1	0.420	0.381	0.313
	0.433	0.413	0.355	2	0.432	0.394	0.331
	0.436	0.418	0.359	3	0.436	0.398	0.335
	0.437	0.418	0.361	4	0.437	0.400	0.337
Mars	0.323	0.209	0.100	1	0.355	0.150	0.077
	0.340	0.236	0.139	2	0.368	0.182	0.120
	0.345	0.242	0.147	3	0.372	0.189	0.128
	0.346	0.244	0.150	4	0.374	0.193	0.131

Assumptions:

1. Equal distribution of ΔV among stages.
2. Propellant fraction = 0.9 for all stages.
3. Zero perturbations.
4. Co-planar transfer.
5. Circular planetary orbits.
6. Instantaneous thrusting for high thrust cases.
7. No mid-course correction required.
8. Orbit altitude at destination is 300 nm.
9. I_{sp} (low thrust) = 800 sec
   I_{sp} (high thrust) = 420 sec

TABLE III-2

PLANETOCENTRIC THRUSTING REQUIREMENTS FOR INTERPLANETARY PROBES

	Vehicle Velocity in 300 n m Orbit	PLANETOCENTRIC THRUSTING (from 300 n m initial orbit)					
		$F_o/W_o = 10^{-3}$ Continuous				High Thrust	
		Burning Time Days	Coast Time Yr	Coast Time Days	ΔV fps	Coast Time	ΔV fps
Earth	24,900						
elliptical to Moon		4.21		3.2	18,900	5 days	9,975
parabolic escape		5.03	--	- -	19,800	- - -	10,310
hyperbolic to Venus		5.6	0.4		23,200	0.4 yr	11,500
hyperbolic to Mars		5.79	0.7		24,200	0.7 yr	11,800
hyperbolic to Mercury		7.29	0.29		38,000	0.29 yr	18,300
hyperbolic to Moon (fast)		4.29			19,200		
Moon	4,780						
parabolic to escape		1.16	--	- -	3,700	- - -	1,980
hyperbolic to Earth		1.54	--	- -	4,350	5 days	2,001
Venus	25,600						
parabolic escape		4.75	--	- -	18,650	- - -	10,600
hyperbolic to Earth		5.44	--	- -	22,600	0.4 yr	11,726
Mercury	8,860						
parabolic escape		2.15	--	- -	6,500	- - -	3,570
hyperbolic to Earth		Note 1	--	- -	Note 1	0.29 yr	25,054
Mars	10,750						
parabolic escape		2.63	--	- -	8,100	- - -	4,450
hyperbolic to Earth		4.00	--	- -	13,600	0.7 yr	6,335

Required hyperbolic velocity could not be attained within Mercury's gravity field by using $F_o/W_o = 10^{-3}$. Heliocentric thrusting is required. Total requirement can be obtained by adding Heliocentric value to Mercury escape value.

TABLE III-3

HELIOCENTRIC THRUSTING REQUIREMENTS

FOR

INTERPLANETARY PROBES

Starting Planet	Helio-centric Orbital Velocity of Planet (fps)	Destination	HELIOCENTRIC THRUSTING REQUIREMENTS			
			$F_o/W_o = 10^{-3}$ Continuous			
			Burning Time (10^{-3} yr)	Total Time (yr)	ΔV (fps)	High Thrust ΔV (fps)
Earth	97,800					
		Venus	7.3	0.46	8,200	8,195
		Mars	7.7	0.7	9,500	9,670
		Mercury	16.0	0.38	25,000	24,741
Moon	3,340	Earth (300 n m alt.)				
		Slow Ellipse				1,047*
		Fast Ellipse				5,139*
Venus	115,000	Earth	NOT COMPUTED			9,098
Mercury	157,200	Earth	NOT COMPUTED			31,554
Mars	78,900	Earth	NOT COMPUTED			7,801

*Earth Centered

TABLE III-4

ΔV AND TIME REQUIREMENTS FOR INTERPLANETARY MISSIONS

LOW THRUST $(\frac{F_o}{W_o} = 10^{-3})$

Mission	Probe			Co-Orbit		Capture	
Destination	ΔV	Coast Time	t_b Days	ΔV	t_b Days	ΔV	t_b Days
Mercury	38,000	0.29	7.29	70,000	13.4	76,500	14.7
Venus	23,200	0.4	5.6	32,298	7.8	45,800	11.05
Moon	18,900	3.2 (days)	4.21	20,000	4.46	23,250	5.19
(fast trip)	19,200		4.29				
Mars	24,200	0.7	5.79	32,001	7.65	42,800	10.22

HIGH THRUST

Mission	Probe		Co-Orbit	Capture
Destination	ΔV	t_{Total} (yr)	ΔV	ΔV
Mercury	19,000	0.29	50,554	54,154
Venus	11,500	0.4	20,598	21,658
Moon	9,975	5.0 Days	11,022	13,022
Mars	11,800	0.7	19,601	24,031

ΔV and Burning Time Requirements

for Interplanetary Missions

(Starting in 300 n m earth orbit)

TABLE III-5

PAYLOADS FOR SHPS WITH STAGED TANKS, IN TYPICAL MISSIONS

$I_{sp} = 800 \quad \eta^{SHPS}_{Tank} = 0.091$

MISSION	ΔV	$(W_u + W_F)$	
		4 tanks	8 tanks
Venus			
Probe	25,300	19,000	19,700
Co-Orbit	34,399	12,400	13,550
Moon			
Probe	19,800	25,200	25,800
Co-Orbit	20,847	24,000	25,000
Capture	24,150	20,000	20,000
Mars			
Probe	26,300	19,000	19,800

Note: Initial Weight = 60 K lb
Thrust-to-weight ratio $F/W = 2.5 \times 10^{-4}$
Current estimate of weight of 15-lb thrust system is 1305 lb

TABLE III-6

% Δ (ΔV) ACCOMMODATED BY SHPS OVER ADVANCED CHEMICAL

ΔV (H.T.) (fps)	Payload Ratio	ΔI_{sp}	$\Delta(\Delta V)$
21K	0.1	800 - 430	19K (90.5%)
12.2 K	0.32	800 - 430	11K (90.1%)
2.8K	0.78	800 - 430	2.4K (85.8%)
600	0.96	800 - 430	500 (83.3%)

TABLE III-7

PAYLOAD RATIO AT THE SAME ΔV FOR SHPS AND ADVANCED CHEMICAL

ΔV (10^3 fps)	P. L. SHPS I_{sp} = 800	CHEM I_{sp} = 420
10	0.63	0.415
20	0.397	0.172
30	0.25	0.071
40	0.158	0.0296
50	0.099	0.0123
60	0.062	0.0051
70	0.039	0.0021

TABLE III-8

ΔV COMPARISON FOR PROBE MISSIONS

Perihelion (10^6 S.M.)	H.T.	SHPS
30	21.5 K	52 K
40	17.5 K	45 K
50	14.8 K	39 K
70	11.7 K	29.2 K

TABLE III-9

ΔV COMPARISON FOR HELIOCENTRIC THRUSTING

(STARTING FROM EARTH ESCAPE)

Perihelion Distribution (10^6 S.M.)	H.T.	SHPS
30	30,000	30,000
40	22,200	22,500
50	16,000	16,300
70	7,000	7,000

IV. RECOMMENDATIONS

The recommendations that can be made as a result of the missions study described in this volume are presented in Volume I.

APPENDIX A

MISSION ANALYSIS HANDBOOK

INTRODUCTION

The objective of this appendix is to provide a rapid, convenient method of synthesizing a space mission; that is, obtaining approximate payload capability of a propulsion system for any space mission (not including launching or landing on a planet). The appendix is divided into three principal sections: Procedure, Mission Requirements, and System Weights and Performance Characteristics. The first section, Procedure, includes the curves relating the basic quantities of the problem (such as ΔV, propellant weight, payload) and a description of how to use the curves to solve a problem. The method and the curves are completely general. The latter two sections include the data that characterize the mission and the system, respectively. The second section, Mission Requirements, needs to be expanded to completely account for the effects of variation in initial thrust-to-weight ratio on the mission requirement. The third section, System Weight and Performance Data, is of course representative of current knowledge; it is subject to change and expansion to include other types of propulsion system and alternate versions of those already included.

The basic quantities represented in the three sections of this appendix are given in Table A-1. Any of the quantities listed in Table A-1 can be obtained as outputs or used as inputs to a problem. The propulsion system types represented are solar-hydrogen, nuclear-hydrogen, and chemical (LOX-hydrogen). The mission requirements section contains ΔV and time data for various space maneuvers. The synthesis of a complete mission consists of combining the requirements of the appropriate maneuvers. The requirements of the mission that must be known include: the planet (or planets) involved; initial and final orbit conditions, such as altitude, inclination, and eccentricity; and attitude requirements. The data of the three sections can be used in a large variety of ways, depending upon which quantities are available as input. The results will be approximate, but are useful for preliminary comparisons of systems and/or missions.

TABLE A-1

IMPORTANT VARIABLES AND SECTIONS OF APPENDIX IN WHICH THEIR VALUES ARE FOUND

SECTION I		SECTION II		SECTION III	
Payload Ratio	$\frac{W_\mu}{W_o}$	Velocity Impulse	ΔV	Propulsion System Weight	$\frac{W_P}{\eta}$
Propellant Ratio	$\frac{W_P}{W_o}$	Condition of Mission	No symbol	Thrust	F
Specific Impulse	I_{sp}	Burning Time	t_b	Propellant Fraction	η
Propellant Fraction	η	System Type	No symbol	System Type	No symbol
Velocity Impulse	ΔV	Transient or Mission Time	t_T	Propellant Weight	W_P
Number of Stages	n				

As an example of the use of this appendix, the relatively conventional approach of selecting a system type, the mission requirements and the initial vehicle weight can be considered. The payload that can be delivered by the selected system type can then be determined in the following manner.

Select the thrust-to-initial weight ratio in Section II for the specified mission, on the basis of minimum ΔV or time allowed. This will yield the ΔV requirement for the mission, and fix the value of thrust, since initial weight is known. The value of ΔV thus obtained can be used in Section I with the known value of I_{sp} to find the propellant ratio (the decimal part of the total initial weight that must be allowed for propellant). Since total initial weight has been given, the propellant weight will then be known. The corresponding propellant fraction can then be obtained in Section III, for the thrust level previously determined. The final payload can then be found in Section I (delivered weight less all weight attributable to the propulsion system). The effect of propulsion system staging can be found from the payload improvement factor curve included in Section I.

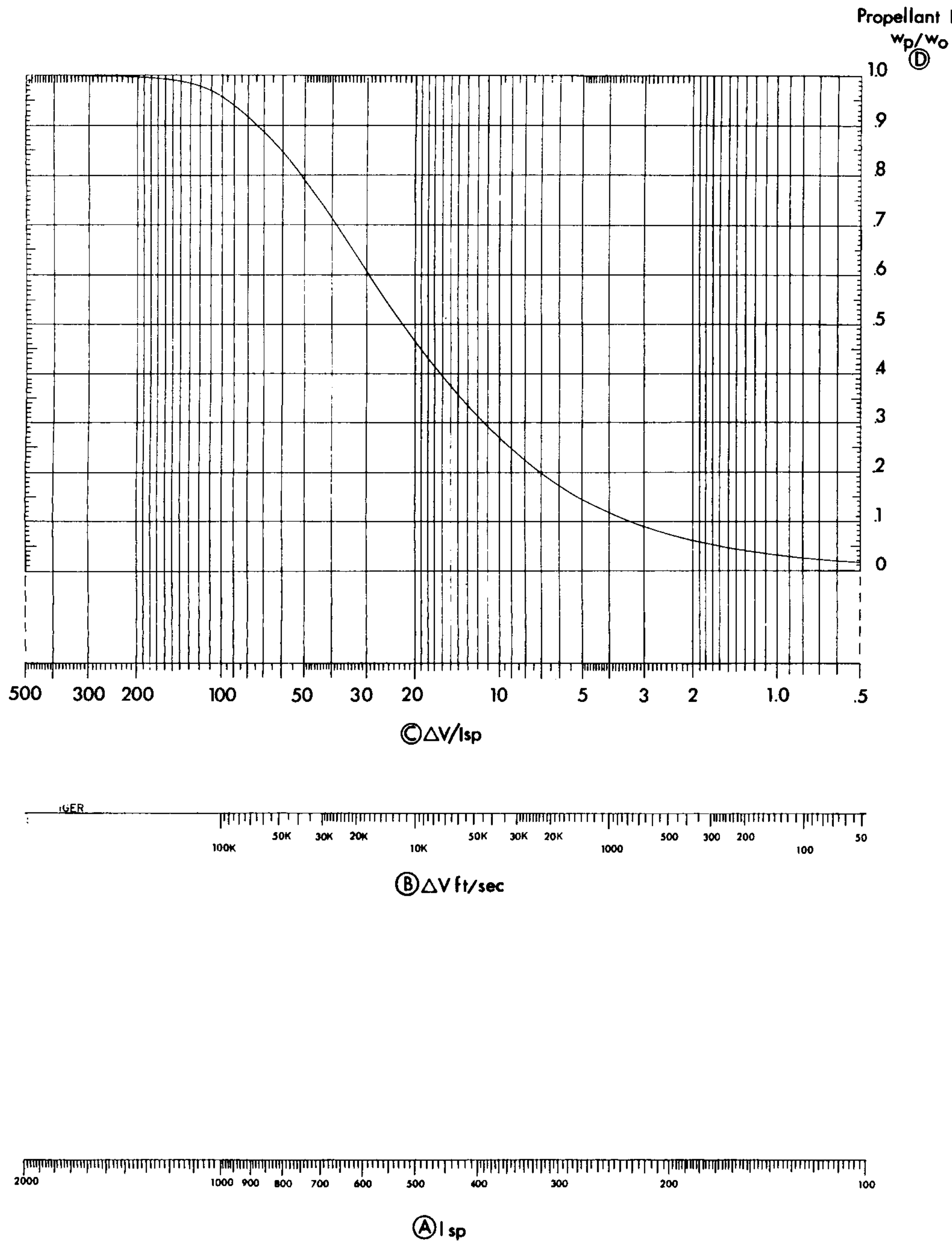
I. PROCEDURE FOR SYNTHESIZING MISSIONS

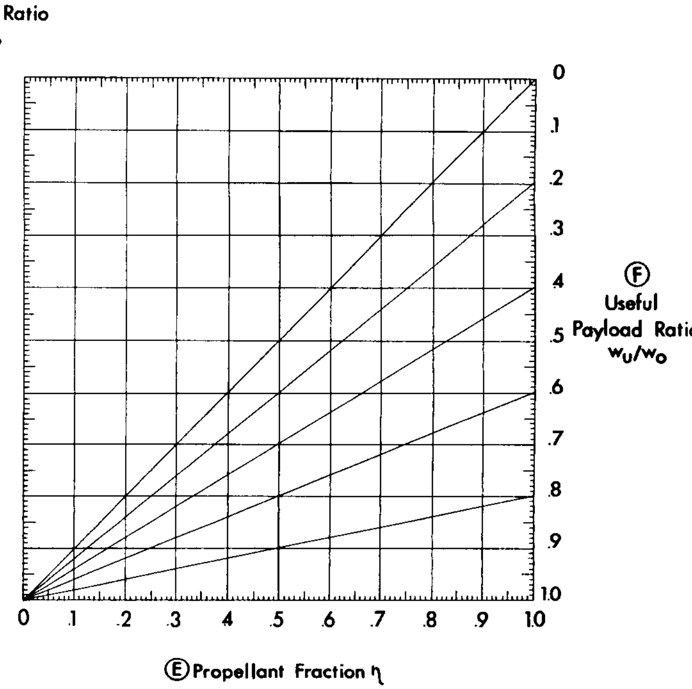
A. MISSION ANALYSIS PROCEDURE

The chart in Figure A-1 can be used to determine the payload capability of any propulsion system with I_{sp} between 100 and 2,000. Values of ΔV between 50 fps and 100 x 10^3 fps per stage can be accommodated. One traversal of the chart applies to a single-stage vehicle. Additional stages simply require additional runs through the chart, the initial weight of a stage being the payload weight of the previous stage. The payload ratio of a series of stages having the same ΔV and propellant fraction can be determined by raising the payload ratio of one such stage to a power equal to the number of stages. An approximate method of obtaining quick knowledge of the effect of staging on payload is to calculate the payload for one stage and multiply by a payload improvement found in Figure A-2.

The procedure for using the chart of Figure A-1 is as follows:

1. Determine the ΔV requirement for the mission from the graphs





PROCEDURE:

1. Select inputs on A and B scales.
2. Find value on C scale.
3. From curve find corresponding value on D scale.
4. Select input on E scale.
5. Pass line through origin and point defined by values on D and E scales.
6. Read Answer on F scale.

DEFINITIONS:

1. W_p is propellant weight (lb).
2. ΔV is ideal velocity (fps).
3. W_o is total initial vehicle weight (lb).
4. I_{sp} is specific impulse $(\frac{lb\text{-}sec}{lb})$
5. W_u is useful payload (arrival weight less empty propulsion system (lb).

FIGURE A-1. GRAPHICAL DETERMINATION OF PAYLOAD RATIO

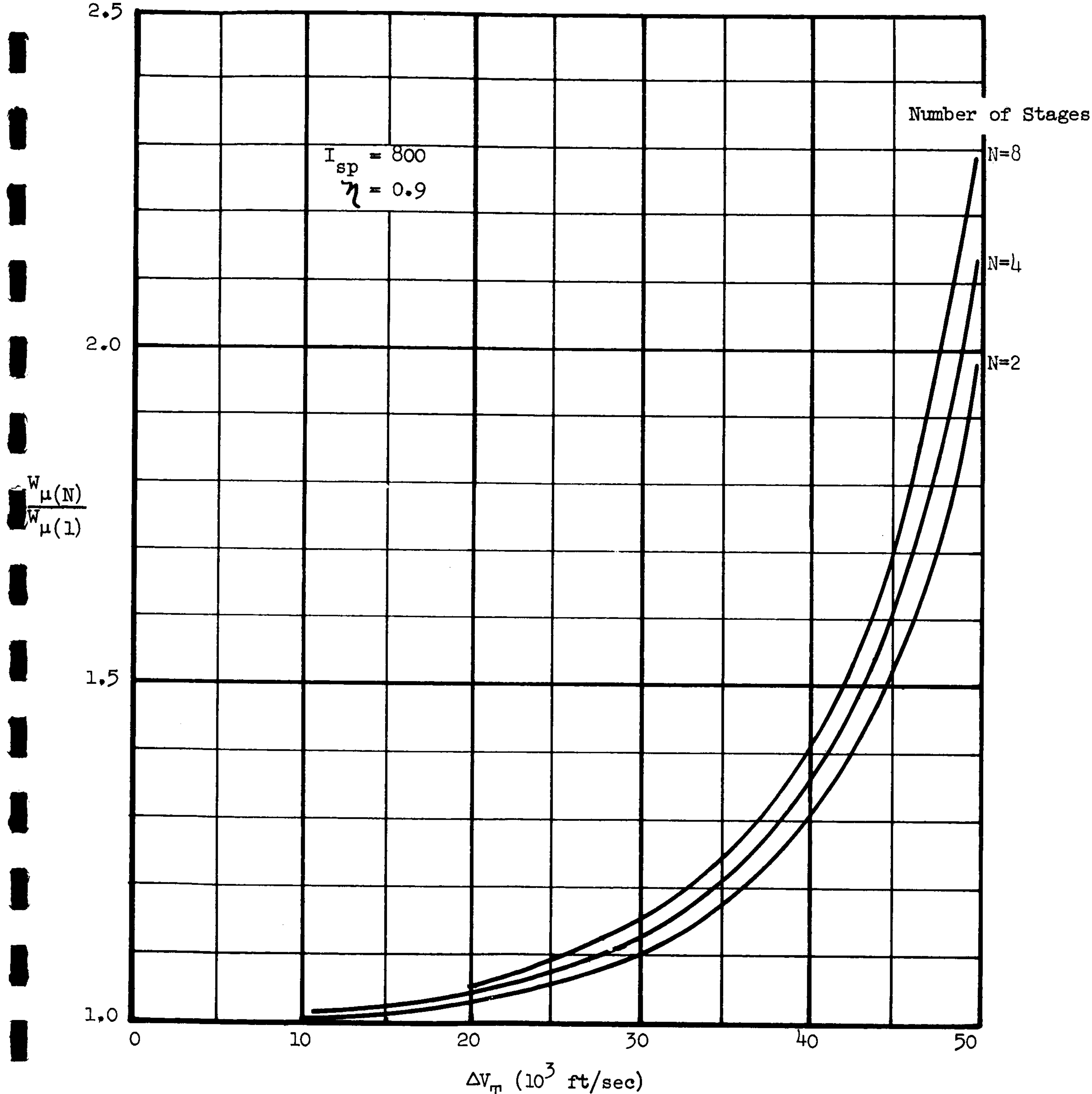


FIGURE A-2. Payload Improvement Factor $\left(\frac{W_\mu(N)}{W_\mu(I)}\right)$ Due to Staging of Section II and find this value on scales B of Figure A-1.

2. Select a value of I_{sp} on Scale A.

3. By means of a straight edge, find the value on Scale C that corresponds to the values of I_{sp} and ΔV already selected.

4. Align straight edge perpendicular to Scale C through the value just found.

5. Place stylus or pencil on the intersection of the straight edge and the exponential curve.

6. Rotate straight edge 90^o. Straight edge will now pass through the value of propellant ratio (W_p/ W_o) on the Scale D. A convenient way of assuring perpendicularity is to make certain that the W_p/W_o) scale reading indicated by the straight edge is the same on the lefthand scale as on the righthand Scale D.

7. Extend the straight edge across the Scale F maintaining the value of W_p/W_o on the Scale D.

8. Select the propellant fraction, η, or determine it from Section III and find that value on Scale E. Place stylus or pencil point on the intersection of the straight edge and the value of propellant fraction.

9. Rotate straight edge until it passes through the zero of Scale E and the intersection.

10. The point of intersection of the straight edge and Scale F corresponds to the payload ratio, W_μ/W_o. This is the ratio of useful payload (arrival weight less all weight attributable to the propulsion system or

accounted for in the selection of propellant fraction, η, to initial weight.

B. SUPPLEMENTARY INFORMATION

The initial weight, W_o, refers to the complete vehicle upon ignition of the stage under consideration. The propellant fraction, η, is the ratio of propellant weight to the sum of propellant and inert weight of the rocket system.

That is

$$\eta = \frac{W_p}{W_p + W_i}$$

The curve of propellant ratio, W_p/W_o, as a function of $\Delta V/I_{sp}$ is the familiar exponential,

$$\frac{W_p}{W_o} = (1 - e^{\frac{-\Delta V}{gI_{sp}}})$$

and the grid containing Scales E and F simply converts from W_p/W_o to W_μ/W_o by

$$\frac{W_\mu}{W_o} = 1 - \frac{1}{\eta} \frac{W_p}{W_o}$$

The chart of Figure A-1 can be used in a number of alternate ways. These ways are shown diagrammatically in Figure A-3. Any quantity in the diagram can be determined if the two quantities in line with it are known or can be found.

Frequently the propellant fraction, η, of a propulsion system is plotted as a function of thrust and propellant weight. To obtain the value of η to use as an input, it will then be necessary to fix one of the vehicle weights: W_o, W_p, or W_μ. Conversely, if the chart of Figure A-1 is used to

ESTABLISHMENT OF VALUES OF ANY TWO VARIABLES IN A STRAIGHT LINE FIXES THE VALUE OF THE THIRD

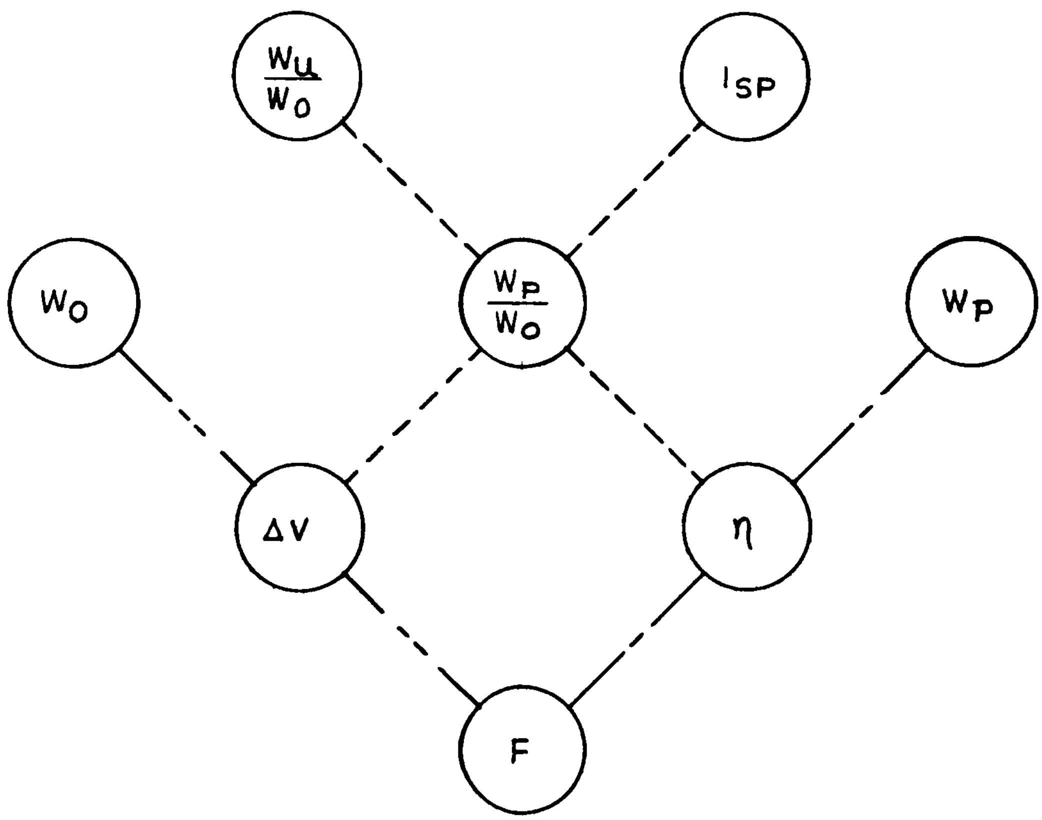


Figure A-3 Diagram of Variables in Mission Analysis produce η as an output, the propulsion system becomes constrained to certain combinations of propellant weight and thrust which are consistent with that value of η. The chart of Figure A-1 is useful for quickly determining the effect of a change in any single variable on the payload ratio or any other variable.

There are other variables of interest in mission-system analysis which are constituents or combinations of the variables represented in Figures A-1 and A-3. They are noted here for convenience:

$\frac{W_p}{\eta}$ The total propulsion system weight

$W_p I_{sp}$ Total Impulse

gI_{sp} Exhaust, or exit, velocity

$W_o - W_p$ Arrival, or burnout, weight

II. MISSION REQUIREMENTS

This section contains graphs showing the ΔV and time requirements for orbital and interplanetary missions. Since ΔV varies with thrust-to-weight ratio in cases for which the assumption of instantaneous impulse is not accurate, three classes of thrusting have been considered:

1. High or instantaneous thrust (thrust acceleration = 0.1 g)

2. Low thrust (thrust acceleration sufficiently low that orbit eccentricity does not change appreciably during missions of interest - thrust acceleration is 10^{-5} g.)

3. Intermediate or moderate thrust (thrust acceleration is 10^{-4} g and 10^{-3} g.)

For the high-thrust data, Hohmann transfers and impulsive velocity changes were assumed. For the low thrust, continuous thrusting with optimized orientation was assumed. Values of ΔV for planetary orbit transfers via this mode were assumed to be equal to the difference between the velocities of the initial and final orbits. For the moderate thrust cases, computer runs were made using numerical integration.

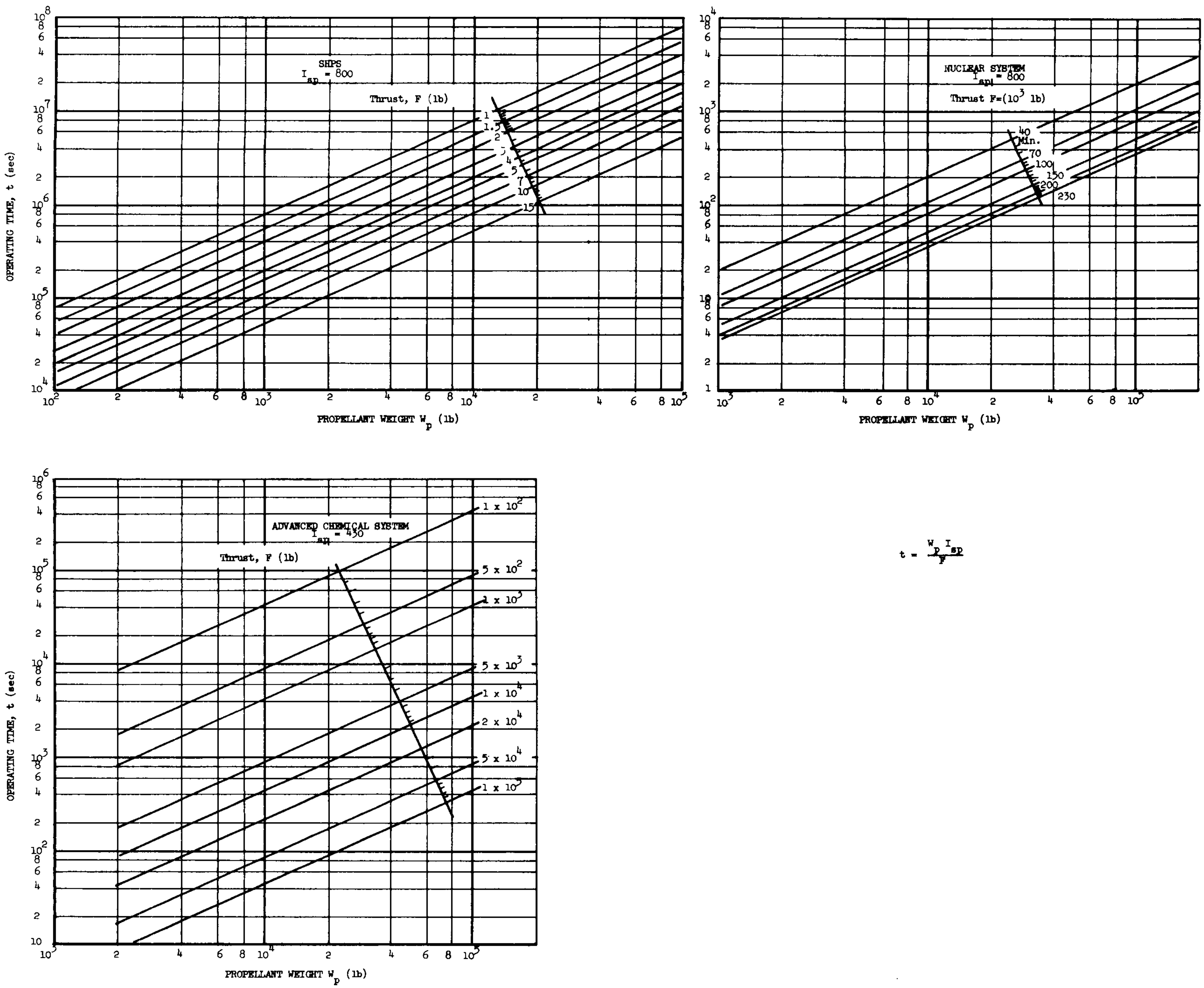
All the types of thrusting described above are not represented in the ΔV data contained in this section. For example, transfer between circular orbits around planets is represented only for high and low thrust. Intermediate thrust is not represented because the saving in transit time realized (over the low-thrust time) by employing the implied higher thrust-to-weight ratios did not seem to warrant the expense of additional computer runs and the circularizing corrections that would have to be added. The solar hydrogen propulsion system can be considered as low or intermediate thrust, depending upon the initial vehicle weight that it is associated with.

In accordance with the above reasoning, the intermediate thrust regime was not included in the calculation of ΔV for the other orbital operations, plane change, eccentricity change, and position change. It was included, however, in the determination of escape requirements, both parabolic and hyperbolic. High-thrust escape was also included but low thrust was not, the ΔV for low-thrust escape being simply the orbit velocity plus the hyperbolic excess. In addition, the time required for escape by low thrust is difficult to define, since it is theoretically achieved at infinite distance from the planet.

Burning times were determined for continuous thrust cases because they are frequently significant in computing total trip time, whereas in high-thrust cases burning time is less significant. Burning time was recorded directly on the graphs for intermediate thrust. For other cases, the plots of time versus propellant weight and thrust in Figure A-4 are provided. The input for Figure A-4 is obtained from ΔV via the chart of Figure A-1 in Section I of this appendix. Assuming a value of I_{sp}, one finds the propellant ratio $\frac{W_p}{W_o}$, which, when divided by the selected value of thrust-to-weight ratio F_o/W_o yields $\frac{W_p}{F_o}$. The value of $\frac{W_p}{F_o}$ so obtained identifies a burning time in Figure A-4 for each of the system types.

The mission requirements are grouped according to the central body with which they are associated. Thus heliocentric operations are in the section labelled "Sun." Numerical integration information (for escape from planets and transfer between planets) is presented only for 10^{-3} g thrust acceleration in the Mars, Venus, Mercury and Moon sections; for 2.5×10^{-4}, 5×10^{-4} and 10^{-3} g in the Earth section; and for the complete range of thrust acceleration represented by thrusts between 1 and 16 pounds and vehicle weights between 10,000 and 200,000 pounds, in the Sun section.

Initial orbital altitude for all numerically integrated escape trajectories was 300 nm. Orbit transfer ΔV for high and low thrust is presented for a wide range of initial altitudes at all the planets of interest in this study.



$$t = \frac{W_p I_{sp}}{F}$$

FIGURE A-4. Operating Time (t_b) vs Propellant Weight (W_p) for Constant Thrust

MISSION REQUIREMENTS
WITH MERCURY AS
THE CENTRAL BODY

FIGURES A-5 THROUGH A-10

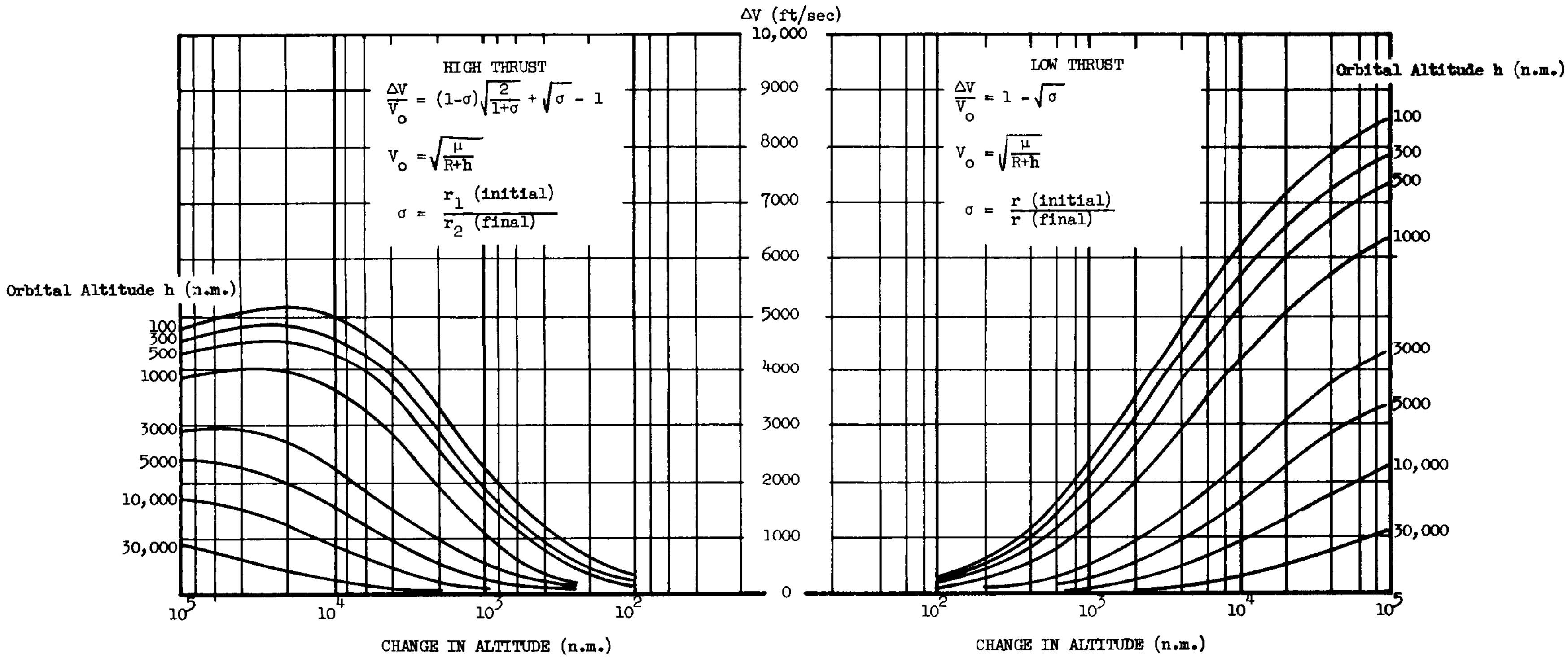


FIGURE A-5. ΔV Requirements for Altitude Change
Mercury Orbits

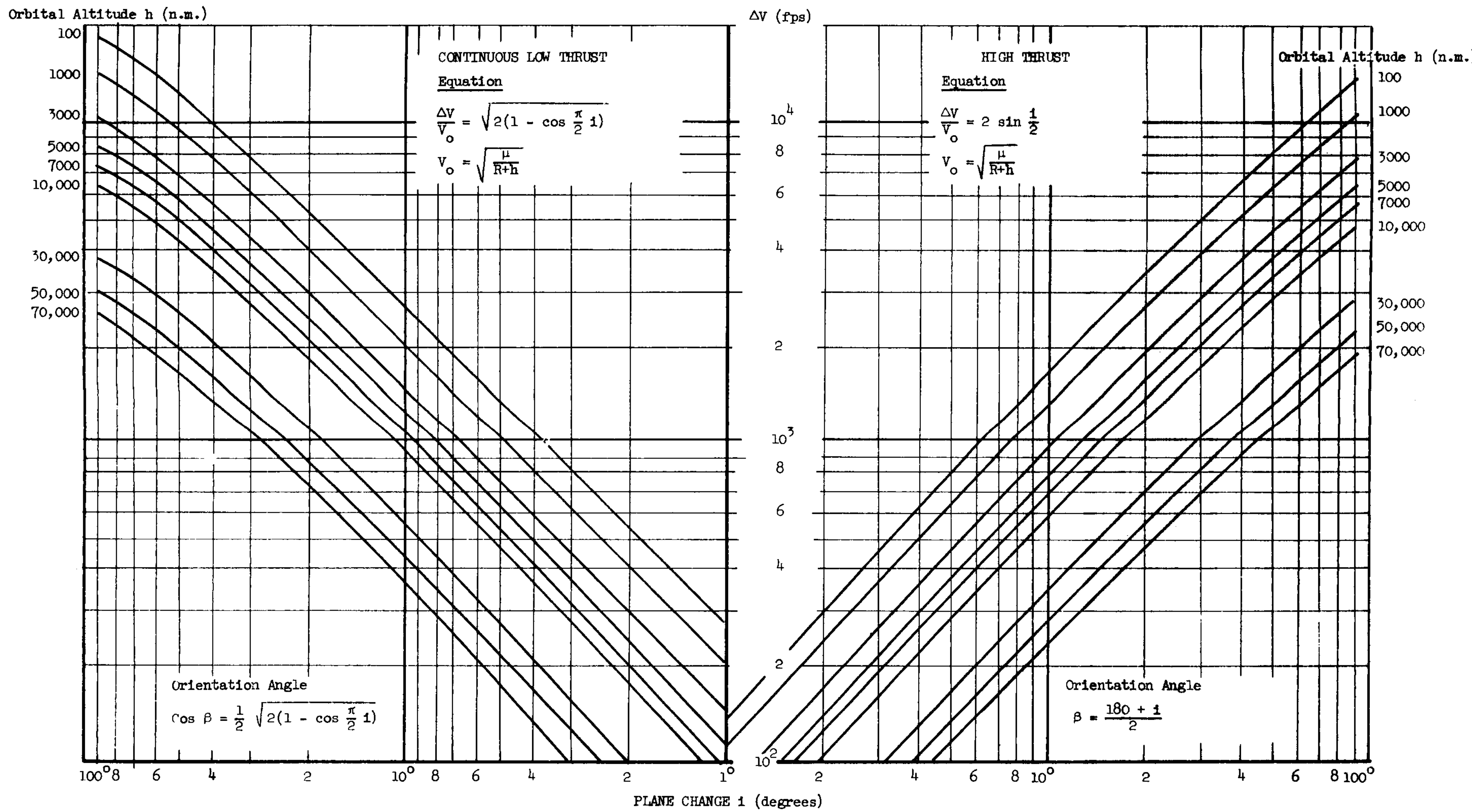


FIGURE A-6. **Plane Change ΔV Requirements**
Mercury Orbit

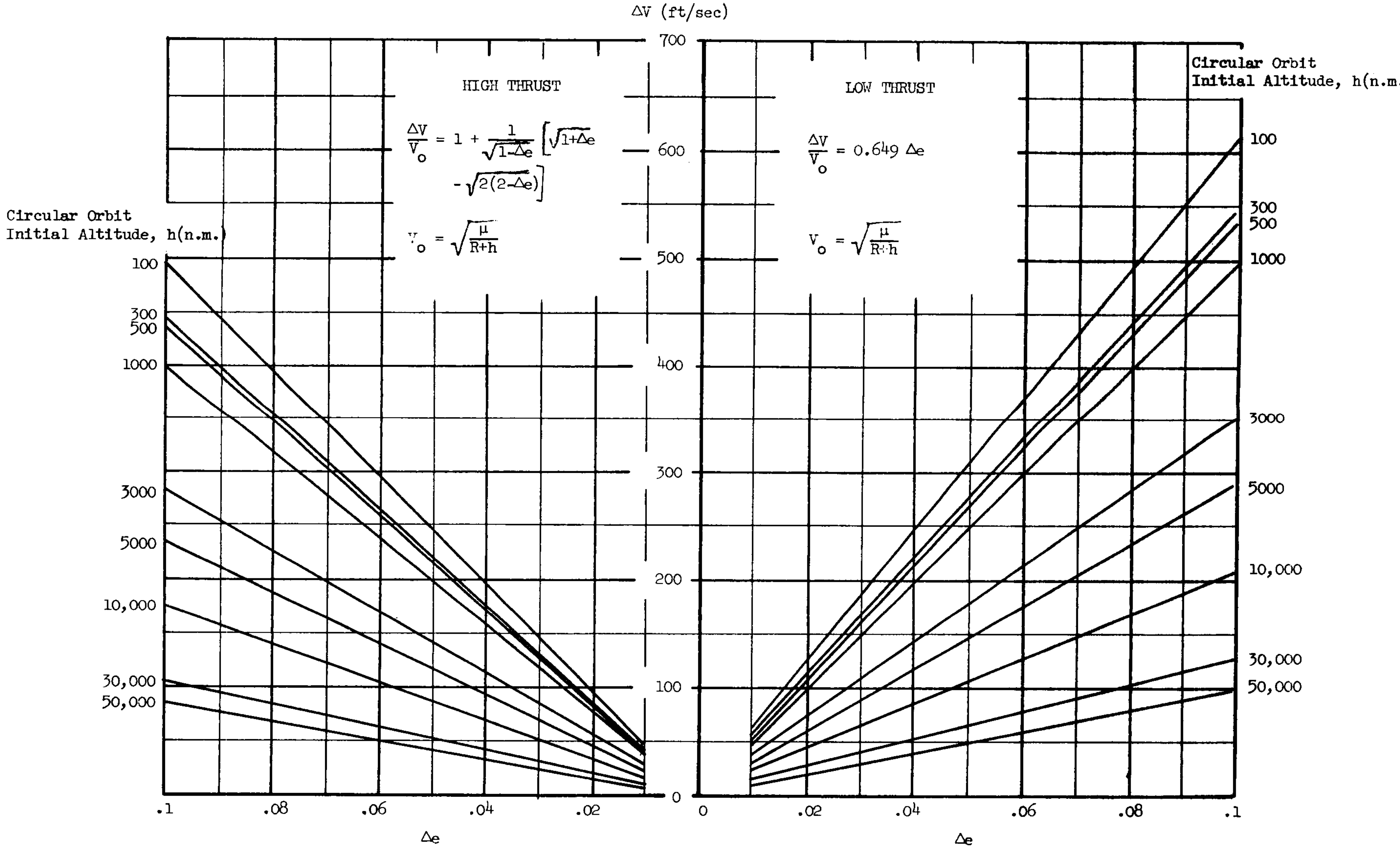


FIGURE A-7. Removal of Orbit Eccentricity
Mercury Orbits

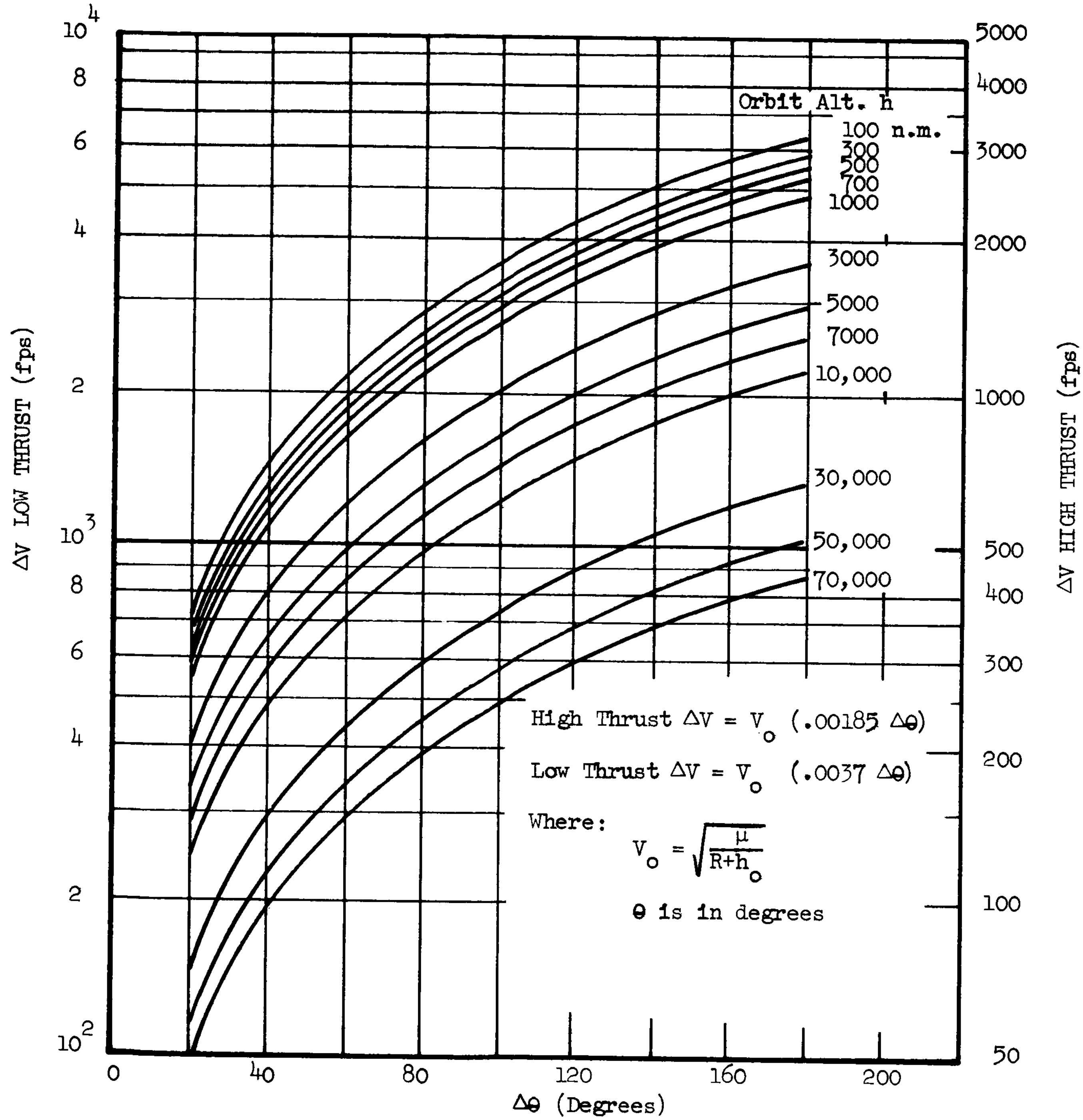


FIGURE A-8. Orbital Position Change Requirements
Mercury Orbits

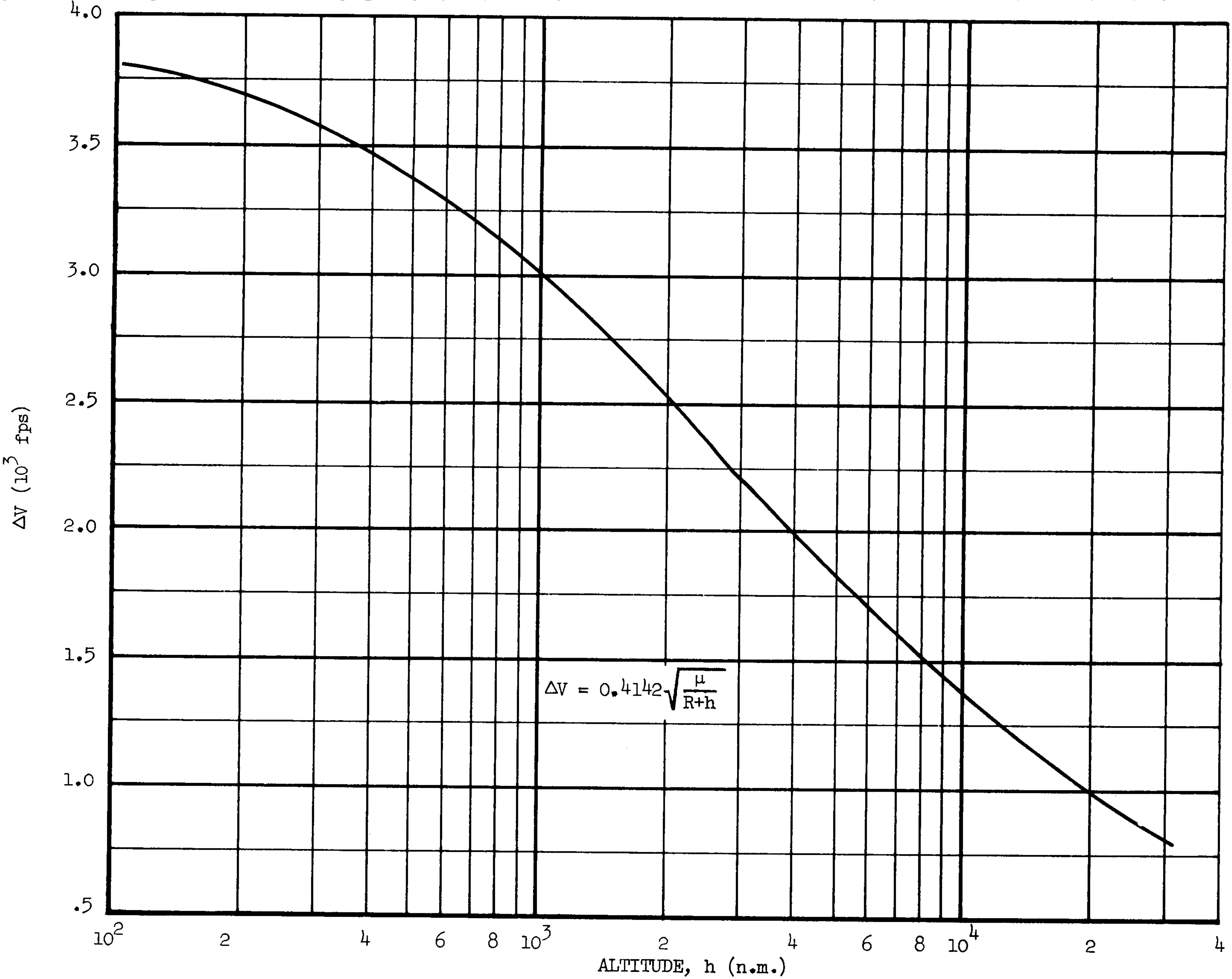


FIGURE A-9. Escape from Mercury Orbit - High Thrust

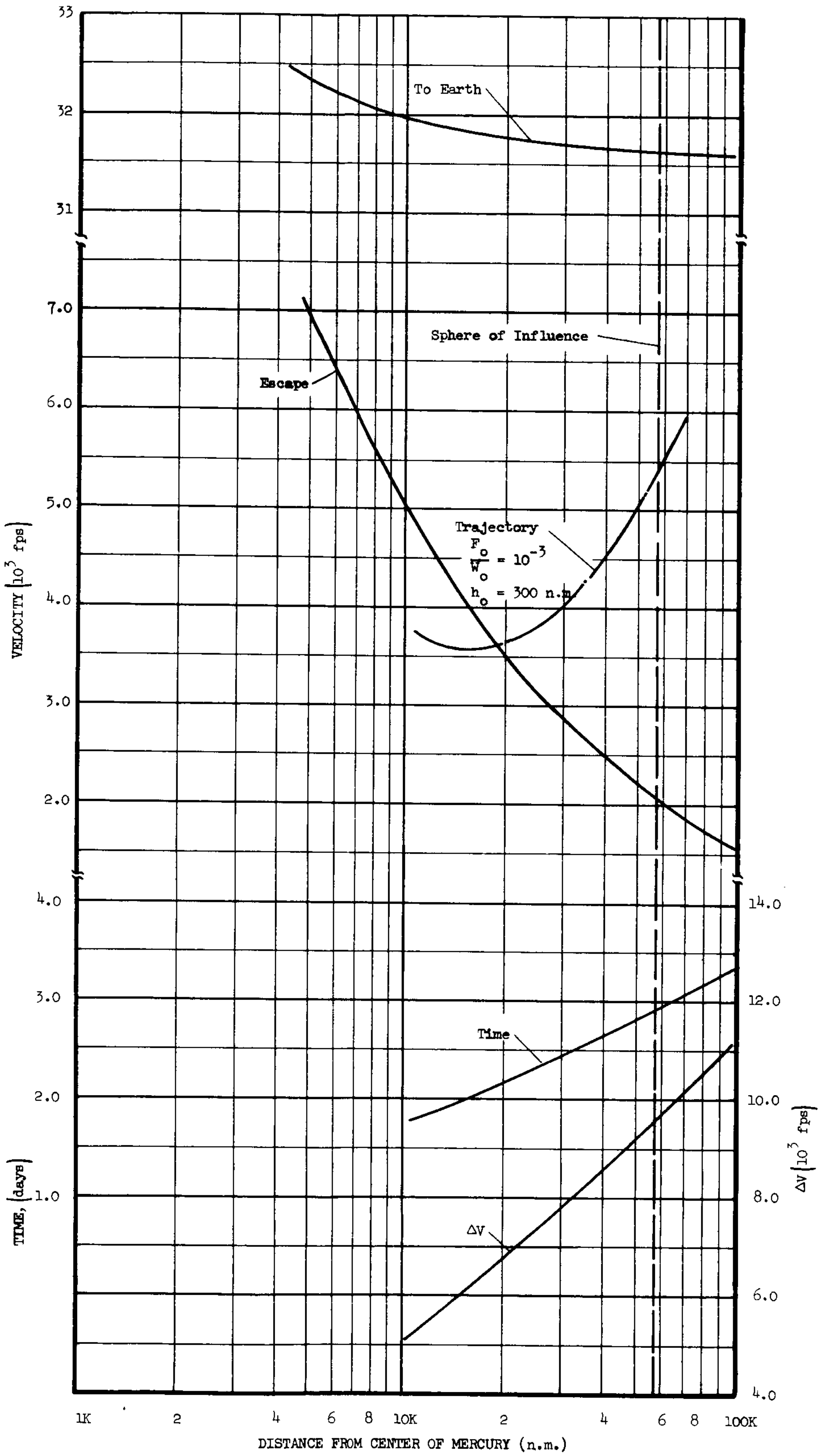


FIGURE A-10. Trajectory and Requirements for Transfer to Earth from Mercury

MISSION REQUIREMENTS
WITH VENUS AS
THE CENTRAL BODY

FIGURES A-11 THROUGH A-16

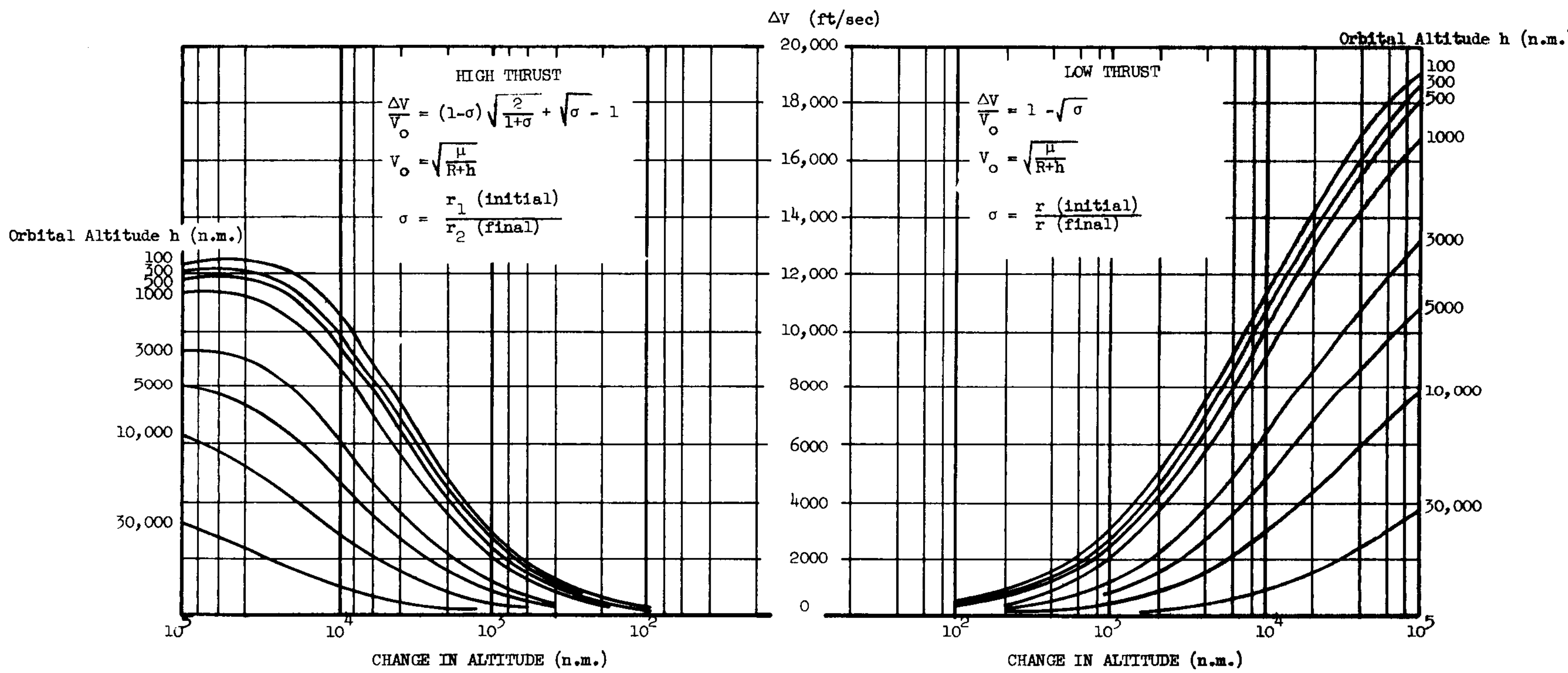


FIGURE A-11. ΔV Requirements for Altitude Change
Venus Orbit

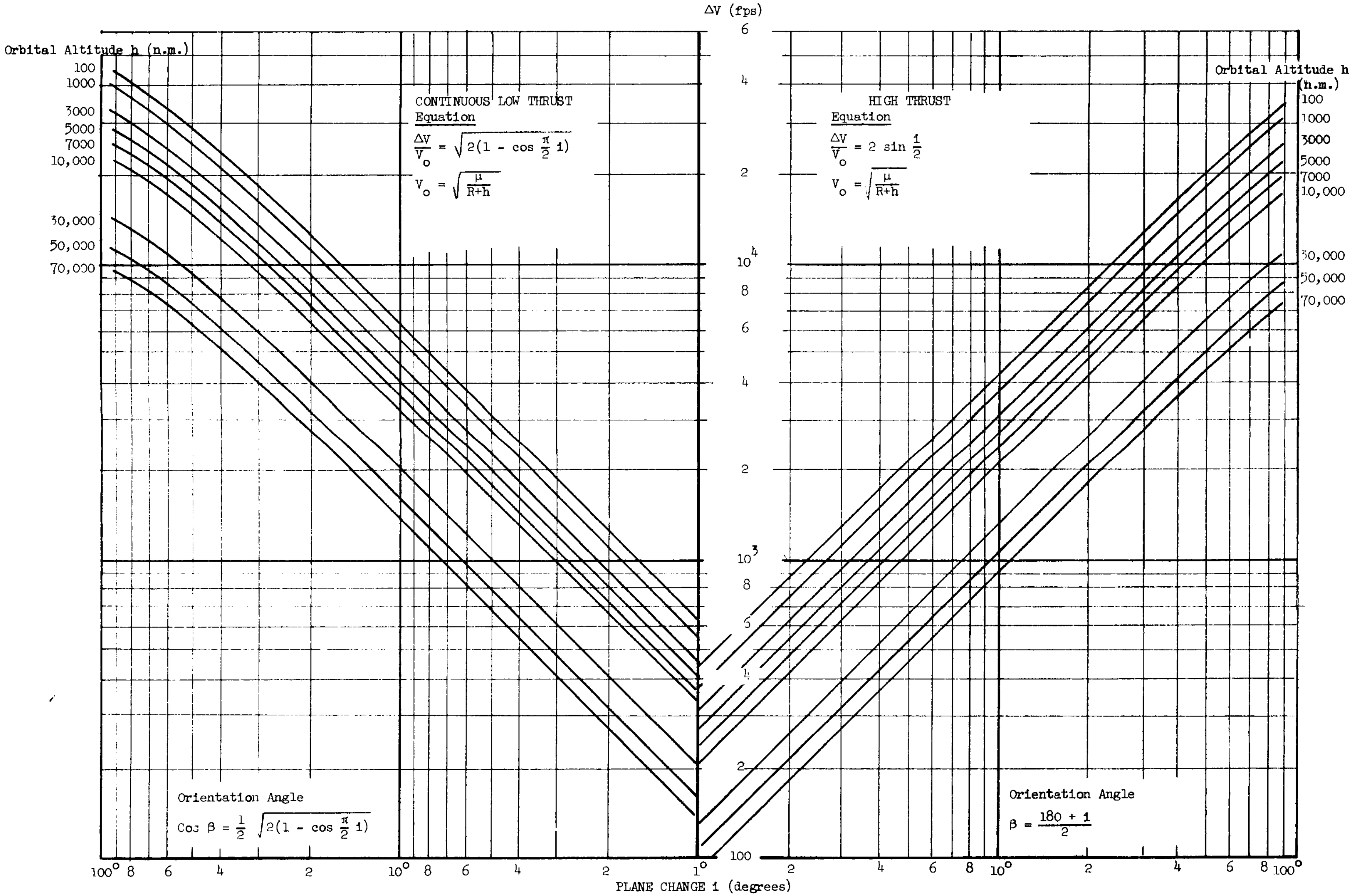


FIGURE A-12. Plane Change ΔV Requirements
Venus Orbit

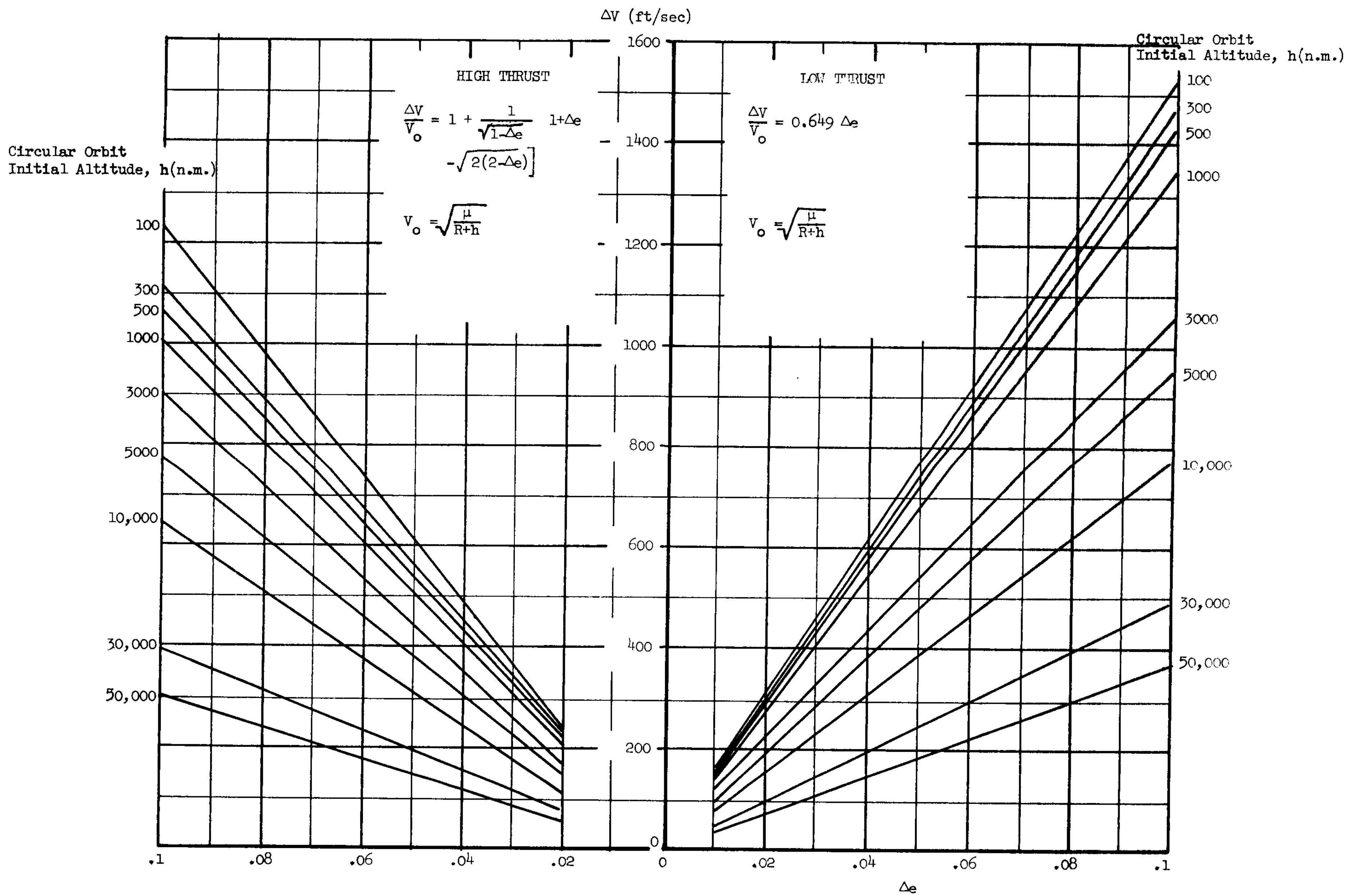


FIGURE A-13. Removal of Orbit Eccentricity
Venus Orbits

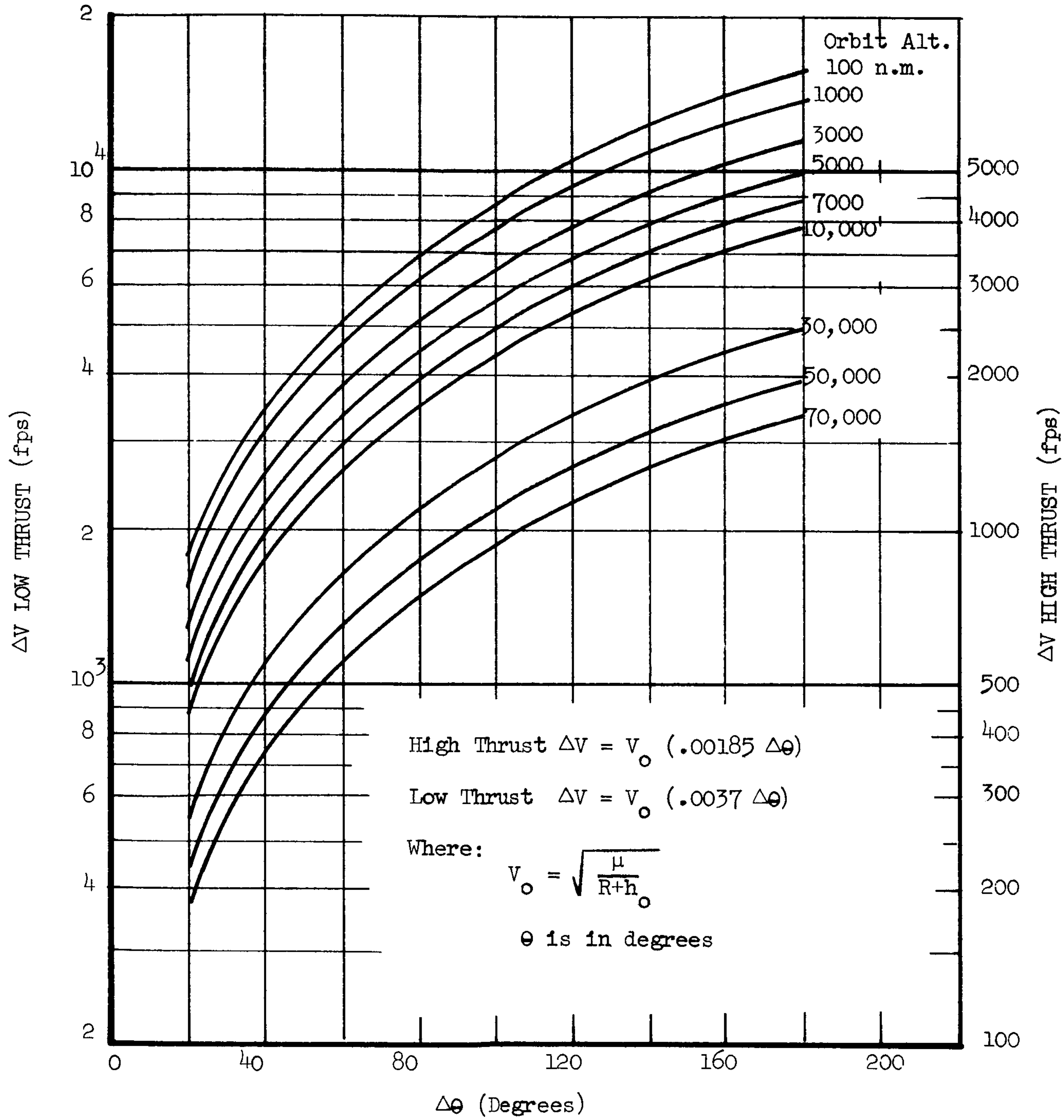


FIGURE A-14. Orbital Position Change Requirements
Venus Orbits

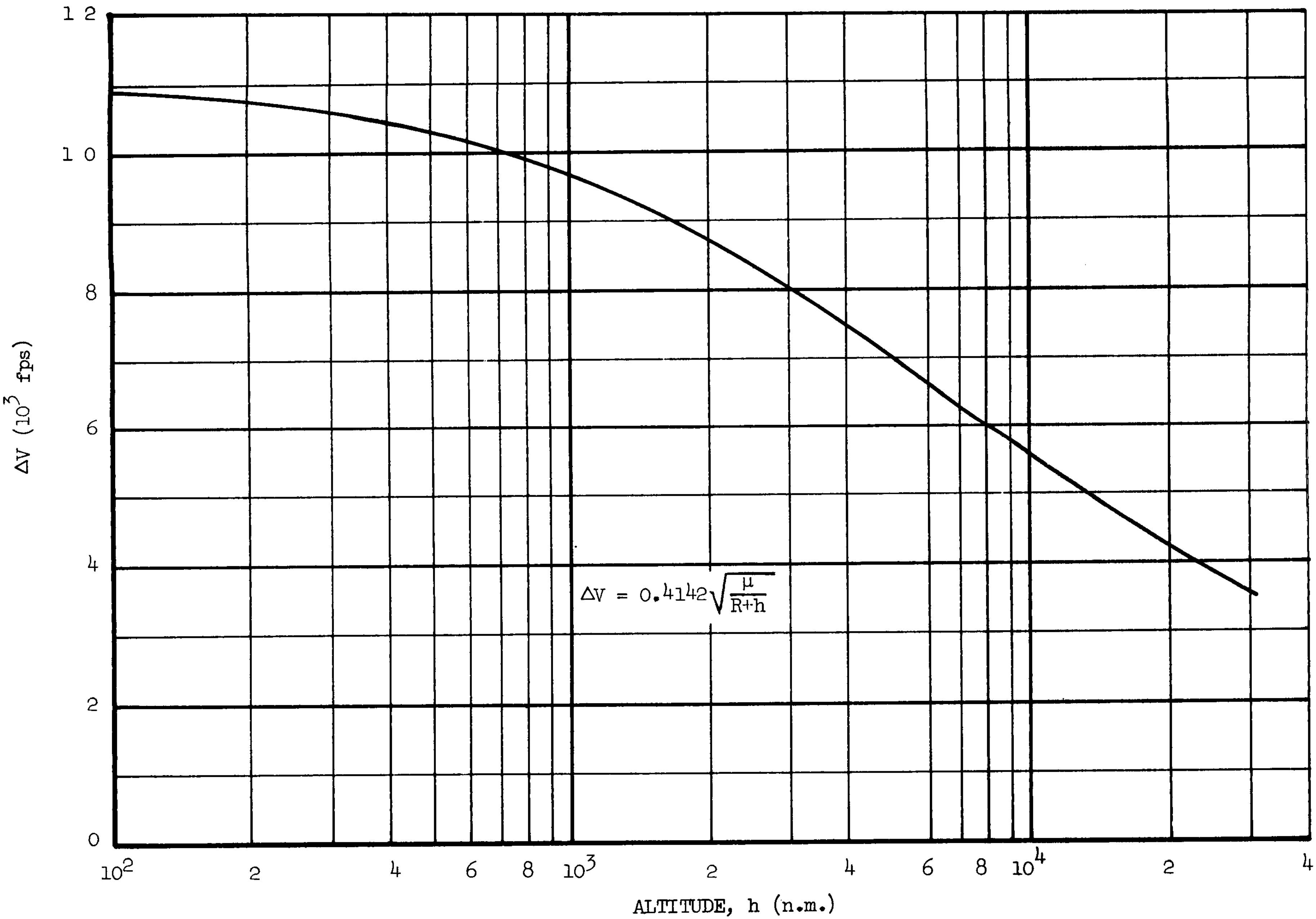


FIGURE A-15. Escape from Venus Orbit - High Thrust

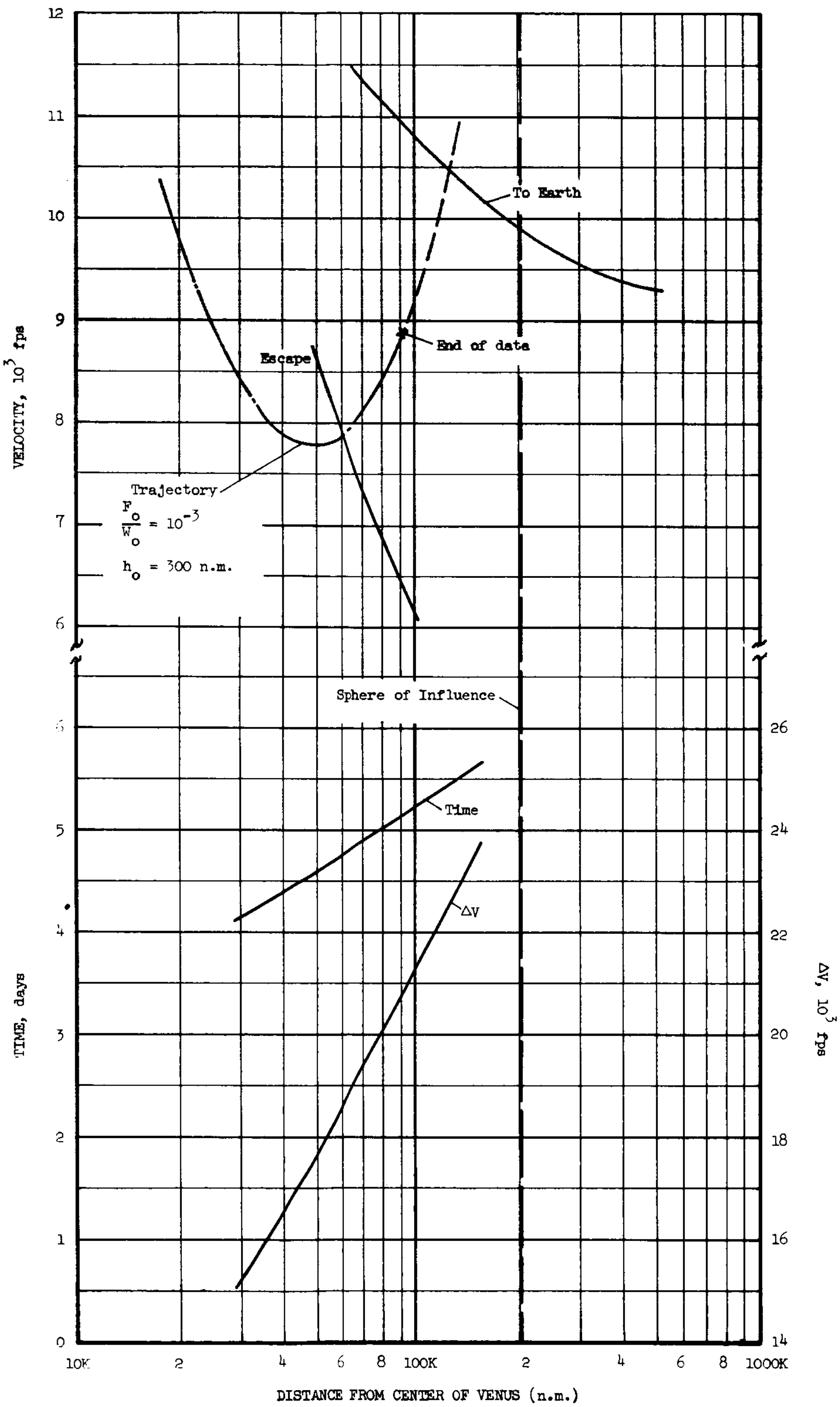


FIGURE A-16. Trajectory and Requirements for Transfer to Earth from Venus

MISSION REQUIREMENTS
WITH EARTH AS
THE CENTRAL BODY

FIGURES A-17 THROUGH A-24

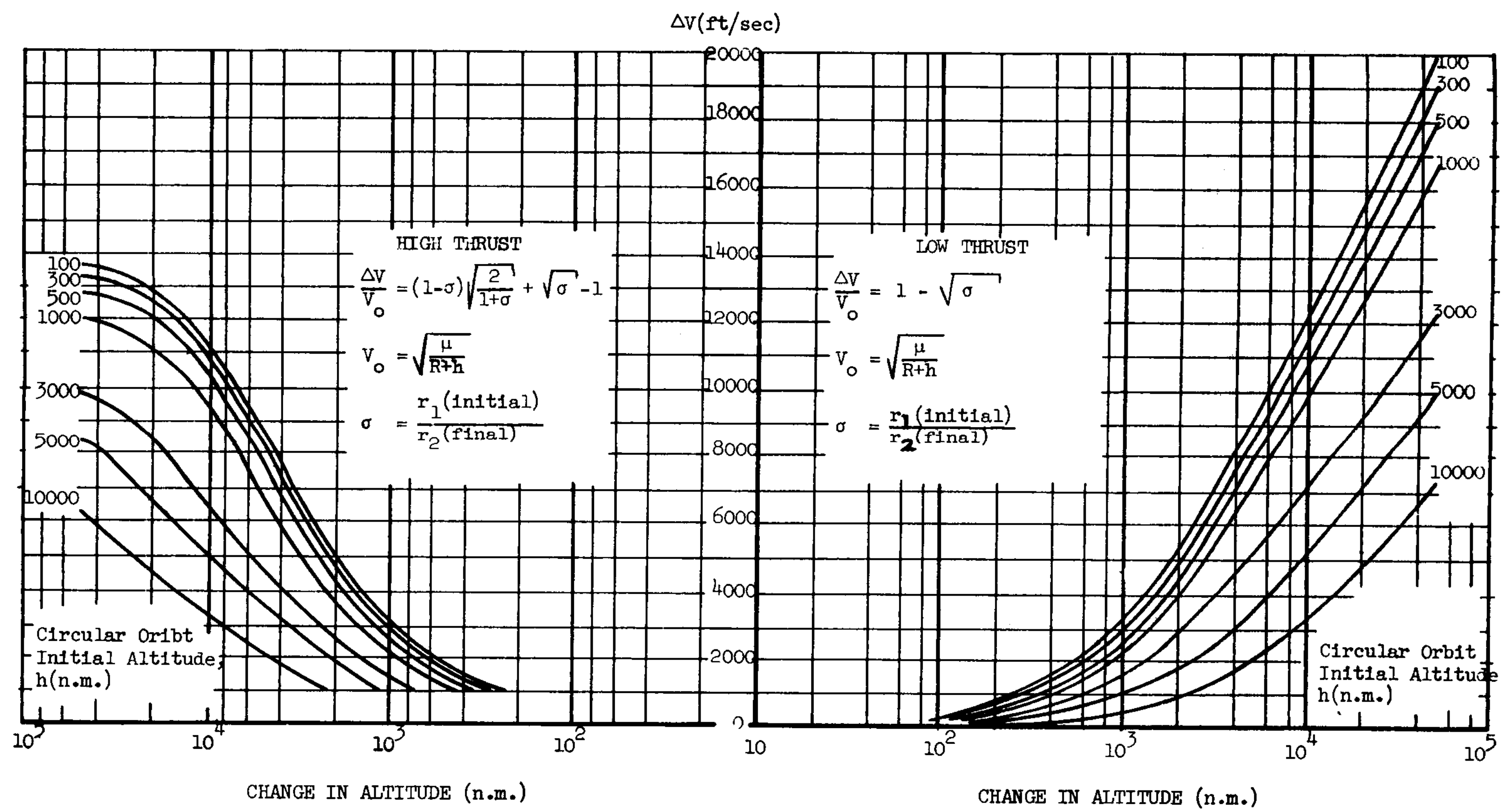


FIGURE A-17. **ΔV Requirements for Altitude Change Earth Orbits**

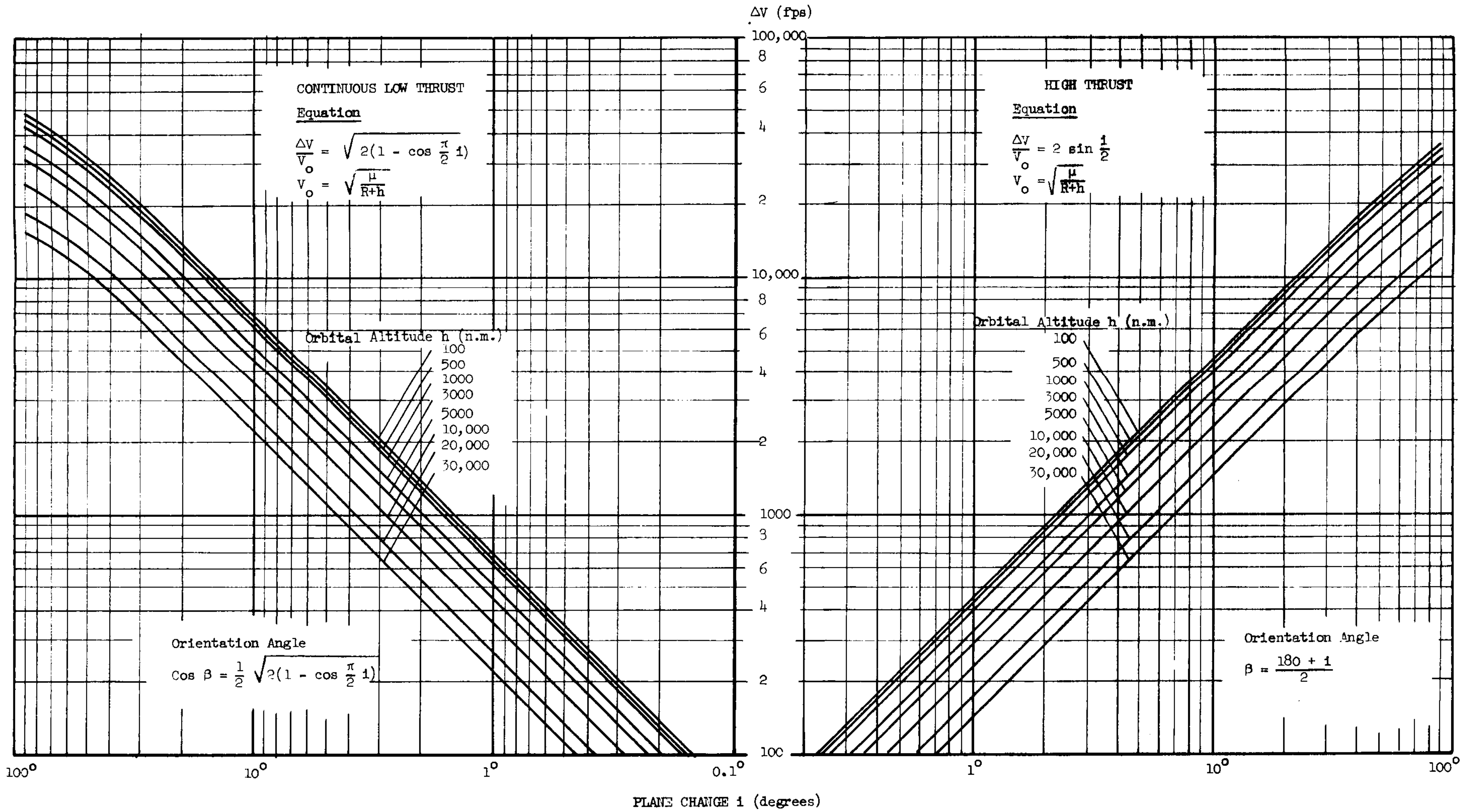


FIGURE A-18. Plane Change ΔV Requirements
Earth Orbit

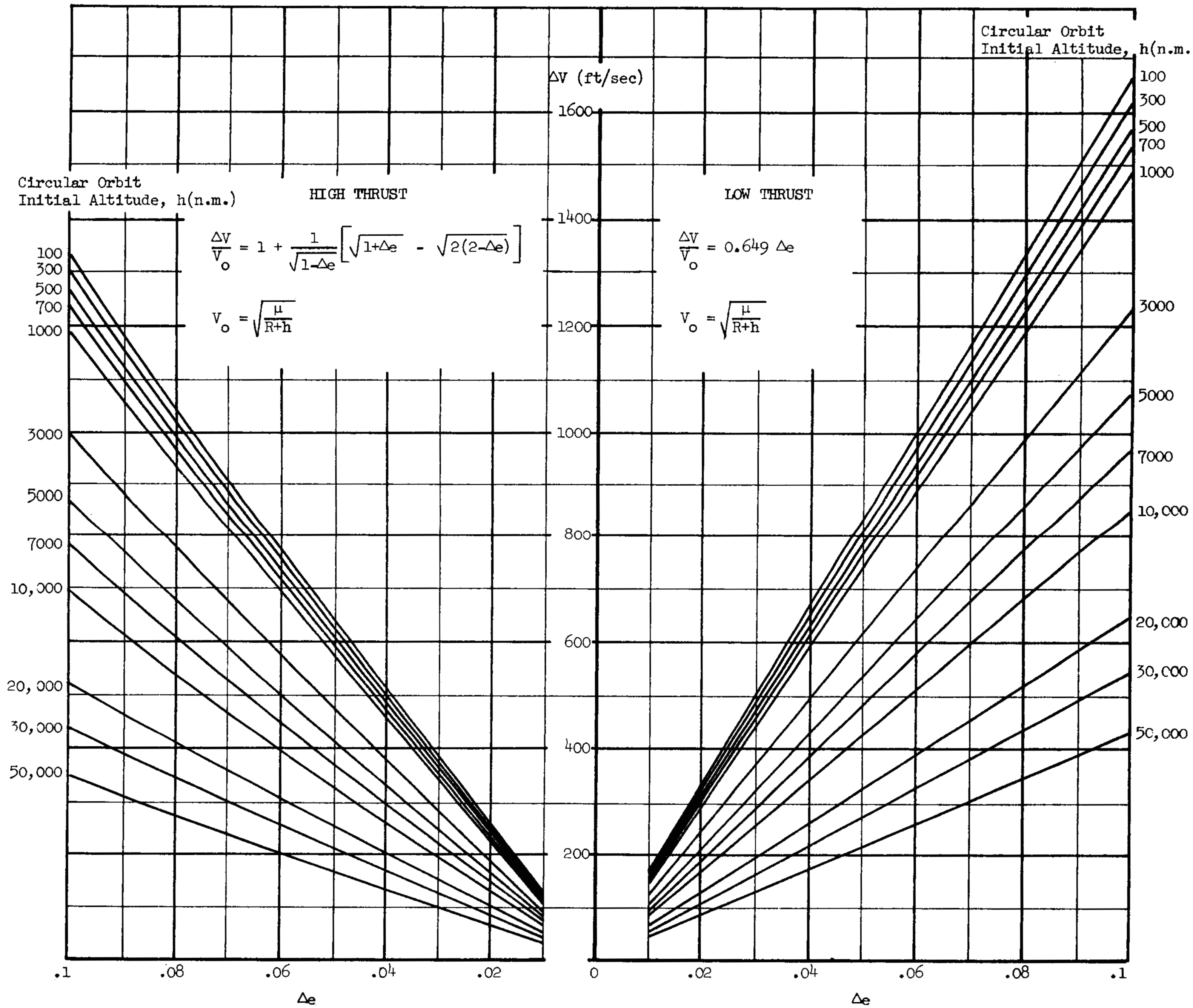


FIGURE A-19. Removal of Orbit Eccentricity
Earth Orbits

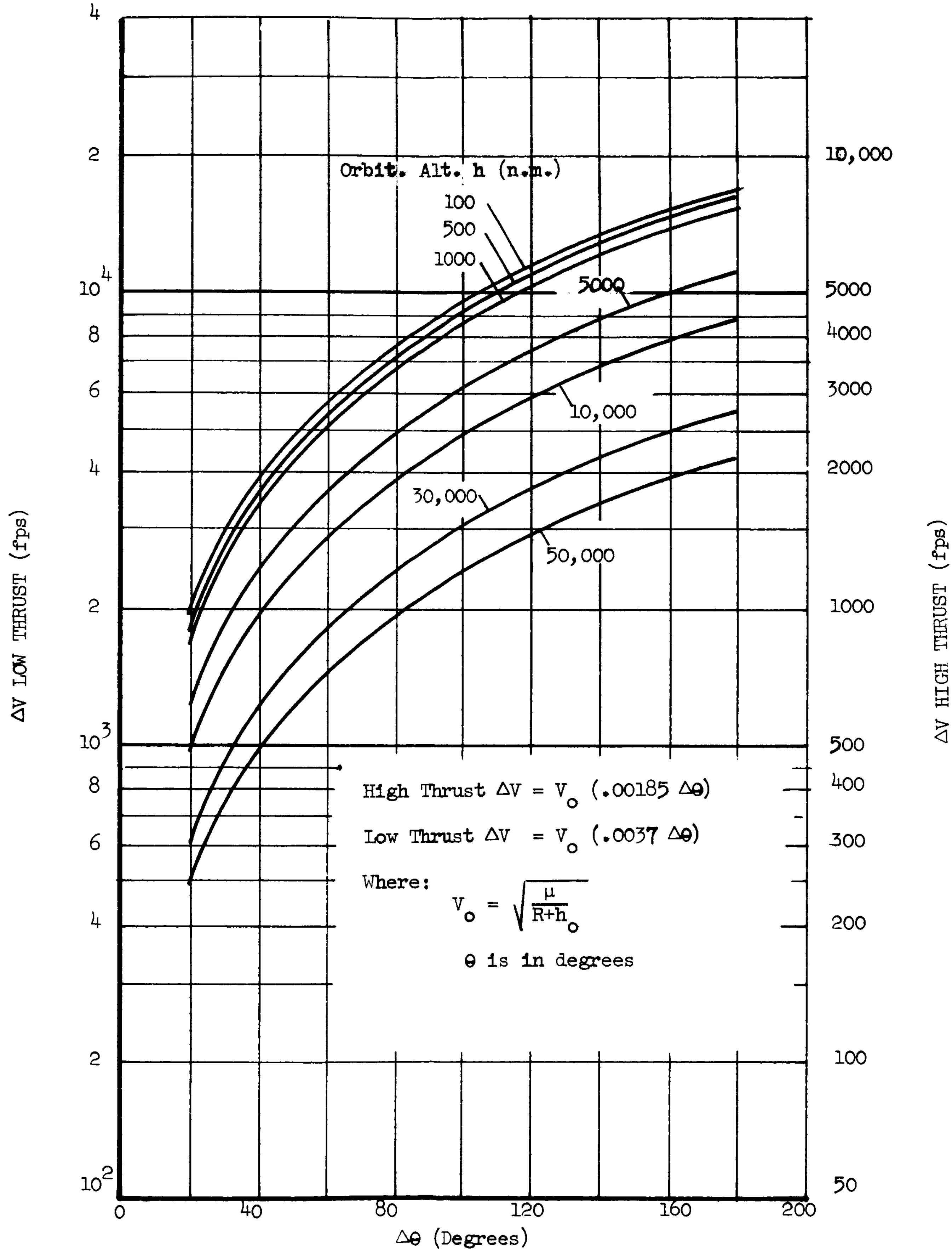


FIGURE A-20. Orbital Position Change Requirements
Earth Orbits

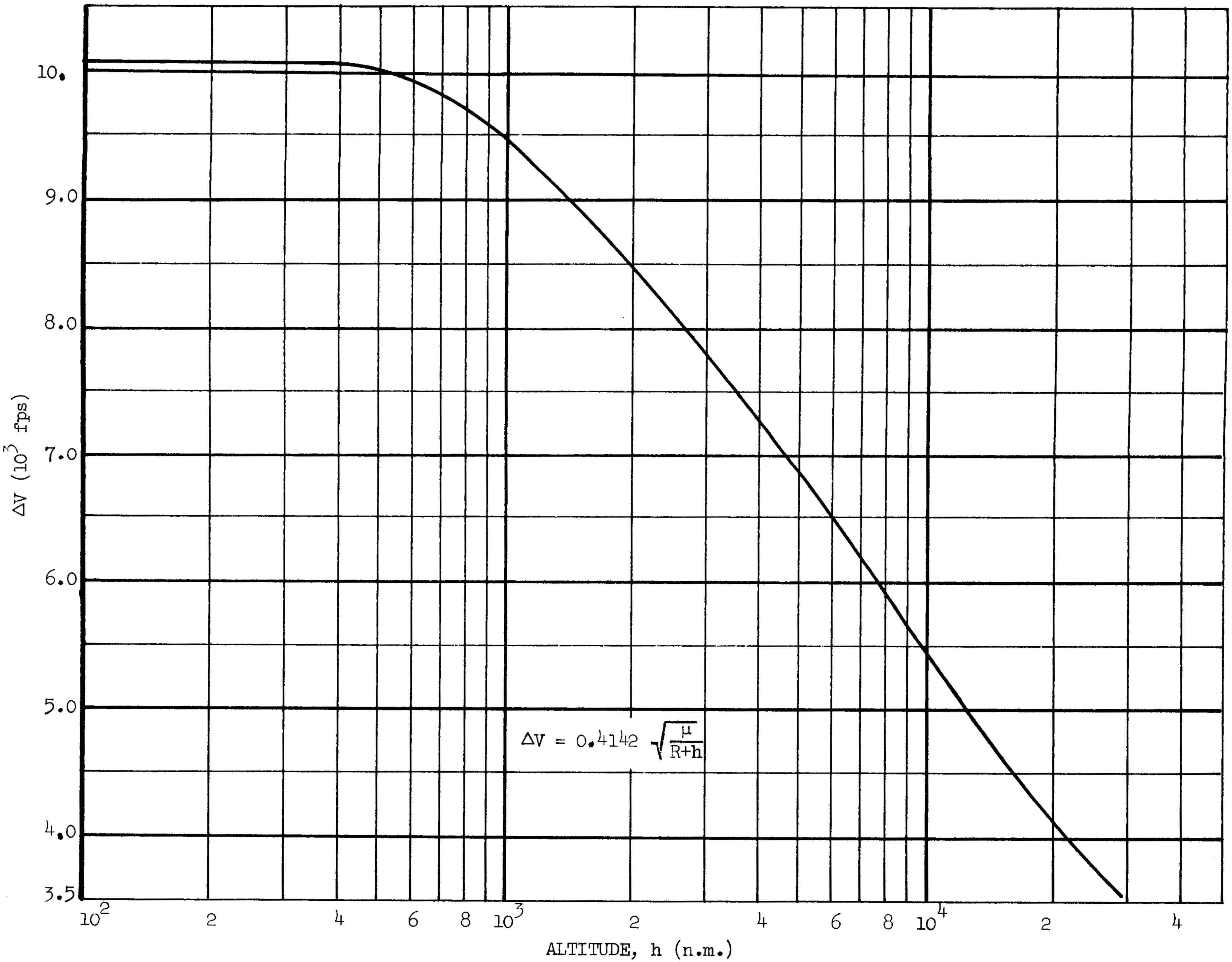


FIGURE A-21. **Escape from Earth Orbit - High Thrust**

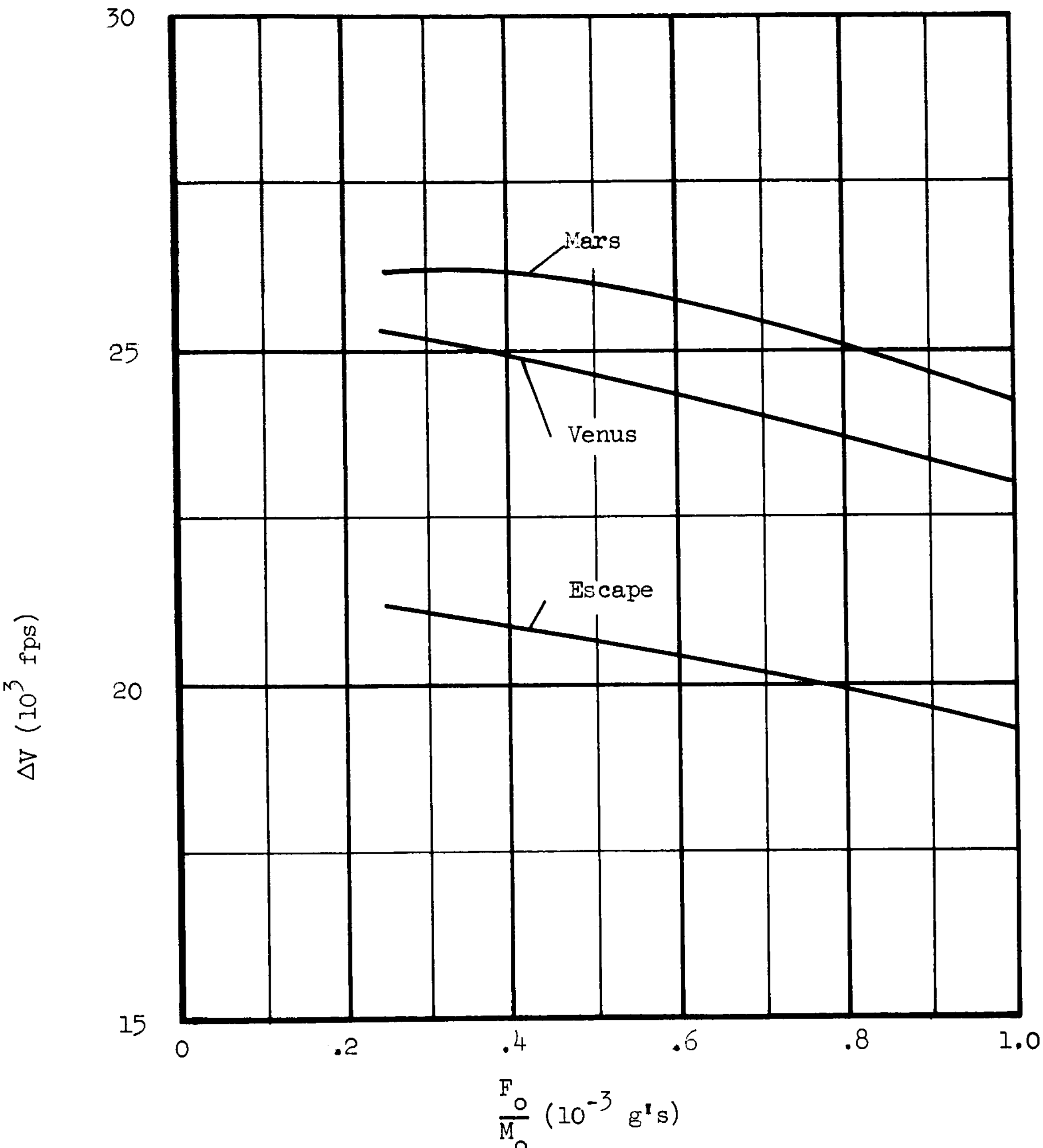


FIGURE A-22. ΔV versus Thrust-to-Mass Ratio $\left(\frac{F_o}{M_o}\right)$ for Departing Earth Orbit (alt = 300 n.m.)

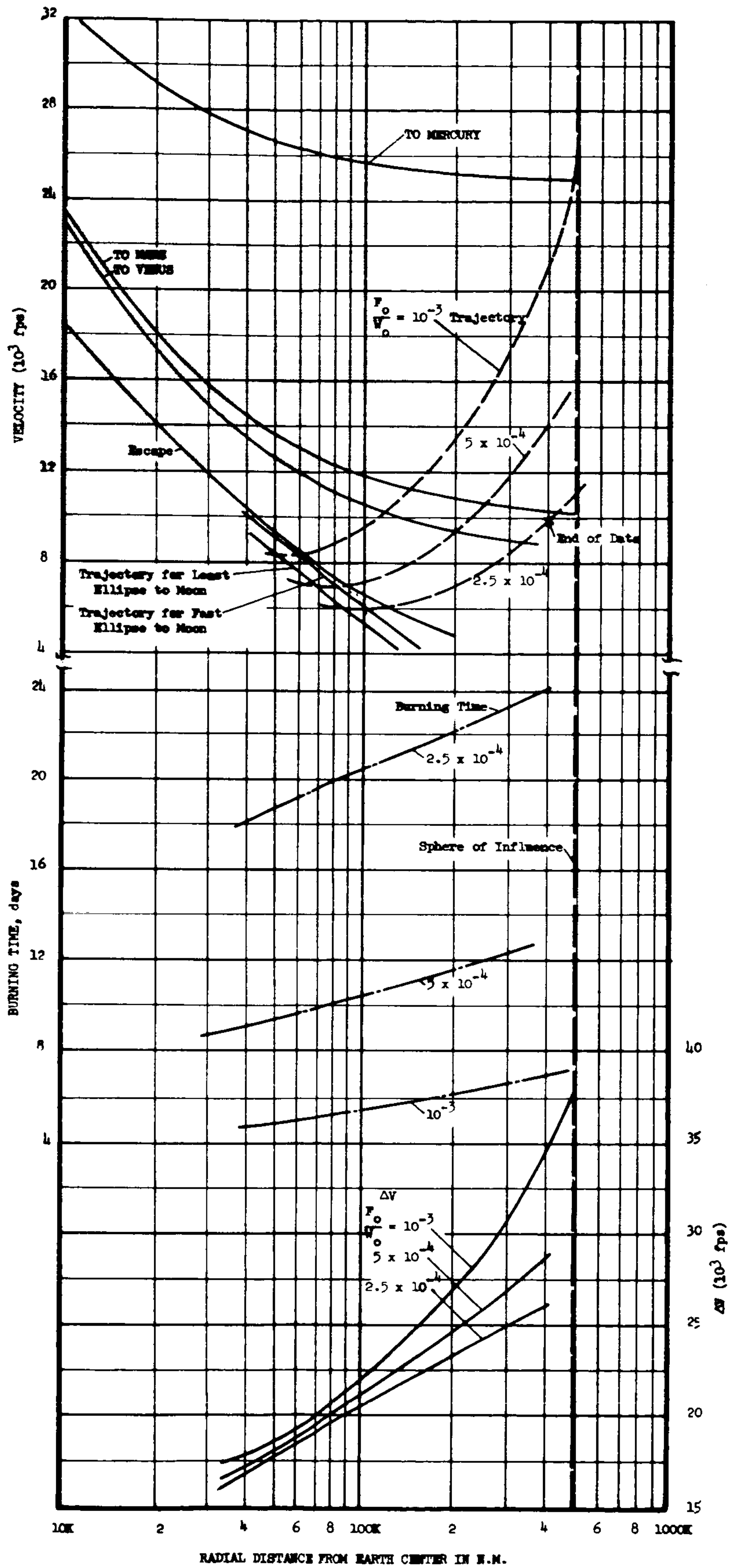


FIGURE A-23. Trajectory and Requirements for Transfer to Other Planets from Earth

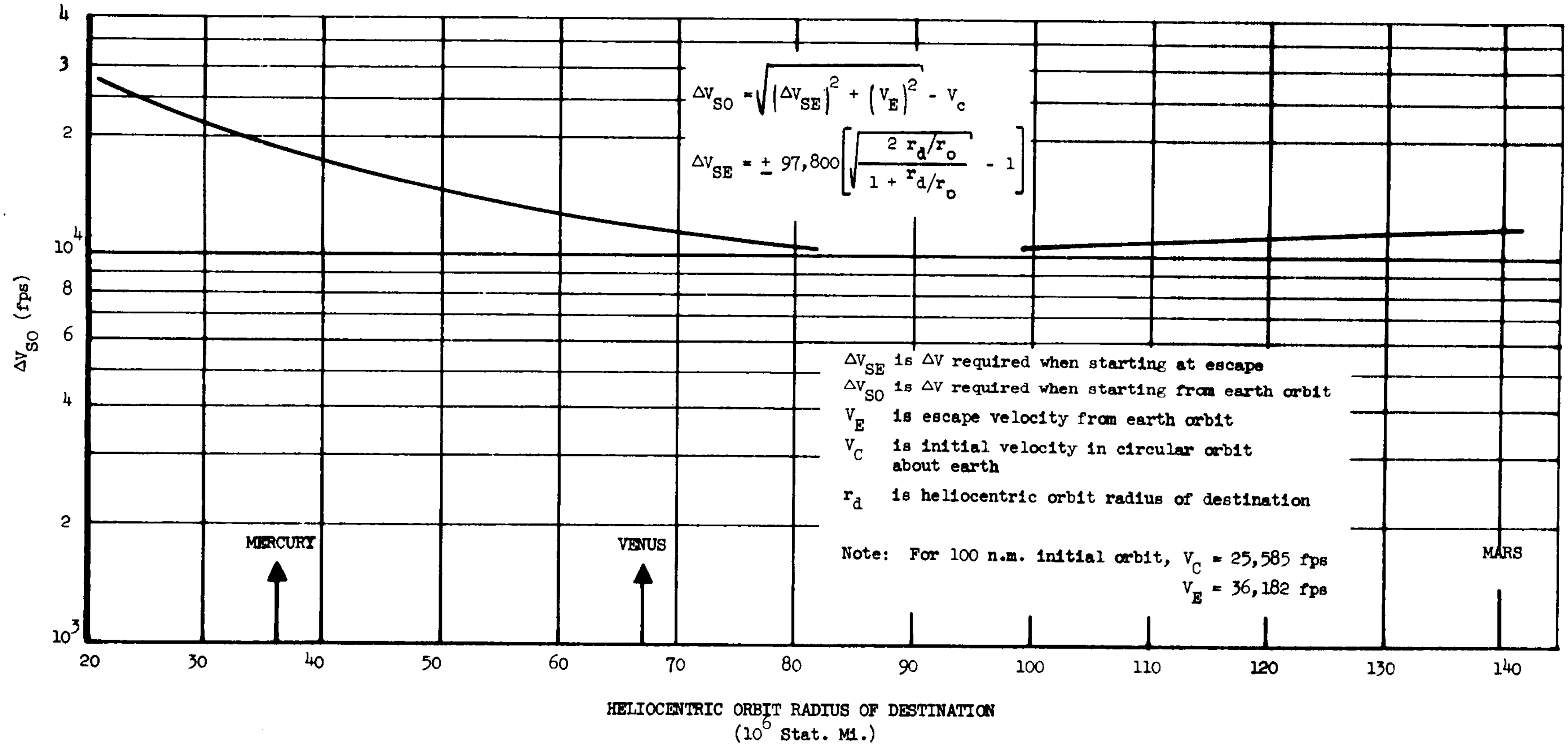


FIGURE . ΔV for High Thrust Probes Starting from 100 n.m. Earth Orbit

FIGURE A-24. ΔV for High Thrust Probes Starting from 100 n.m. Earth Orbit

NOTE ON EARTH ESCAPE DATA

Results of trajectory optimization studies generally suggest two rules which have come into practical use in propellant conservation for space vehicles. These rules can be expressed analytically, but in simple terms are:

(1) Avoid thrusting against gravity.

(2) Attempt to create conditions in which the thrusting may be applied along the velocity vector.

In accordance with the above, the ideal condition exists when the velocity vector is normal to the gravity vector, as it is in a circular orbit. In such a case, circumferential or tangential thrusting (which are equivalent for a circular orbit) is most efficient and radial thrusting is least efficient. This conforms also to the theory of Hohmann transfers, in which thrusting is always performed at the apsides of the transfer orbits.

Computer data obtained in this study from runs made in accordance with the above shows that the minimum ΔV requirement for any of the space-probe missions starting from Earth orbit is obtained by employing the highest possible thrust-to-weight ratio. Earth escape trajectories were run using three different thrust-to-weight ratios and these trajectories compared with velocity-distance requirements for simple escape and hyperbolic flights to other planets. The ΔV for a Venus probe, for example, was found to behave approximately in accordance with the equation

$$\Delta V = \left(- \frac{10^4}{3} \frac{F_o}{W_o} + 26.2\right) \times 10^3$$

The quantities which affect the variation of ΔV with thrust-to-weight ratio were not studied during this project. Sufficient time or funds were not available to make runs with different starting altitudes or to try different F/W ratios in escaping Mercury, Mars and Venus. Further studies of this sort would provide a map of conditions which contribute to reduction of ΔV in the continuous thrust regime for fixed-thrust orientation program.

The variation of ΔV with thrust-to-weight ratio was plotted only for Mars, Venus and parabolic escape trajectories. The requirements for hyperbolic escape to Mercury were such that they could be achieved only with F_o/N_o of 10^{-3} within the Earth's sphere of influence. Therefore, a period of heliocentric thrusting must be matched to the conditions obtained at the sphere of influence to obtain the requirements for hyperbolic escape to Mercury using the 0.5 and 0.25×10^{-3} g acceleration levels. This was not done. The use of a trajectory program which switches central bodies and includes perturbations would be more effective in handling problems of this nature. Such a program exists at Space-General but lacks some of the other characteristics that were needed for this study. It is expected that these differences will be resolved in the current year.

MISSION REQUIREMENTS
WITH MOON AS
THE CENTRAL BODY

FIGURES A-25 THROUGH A-30

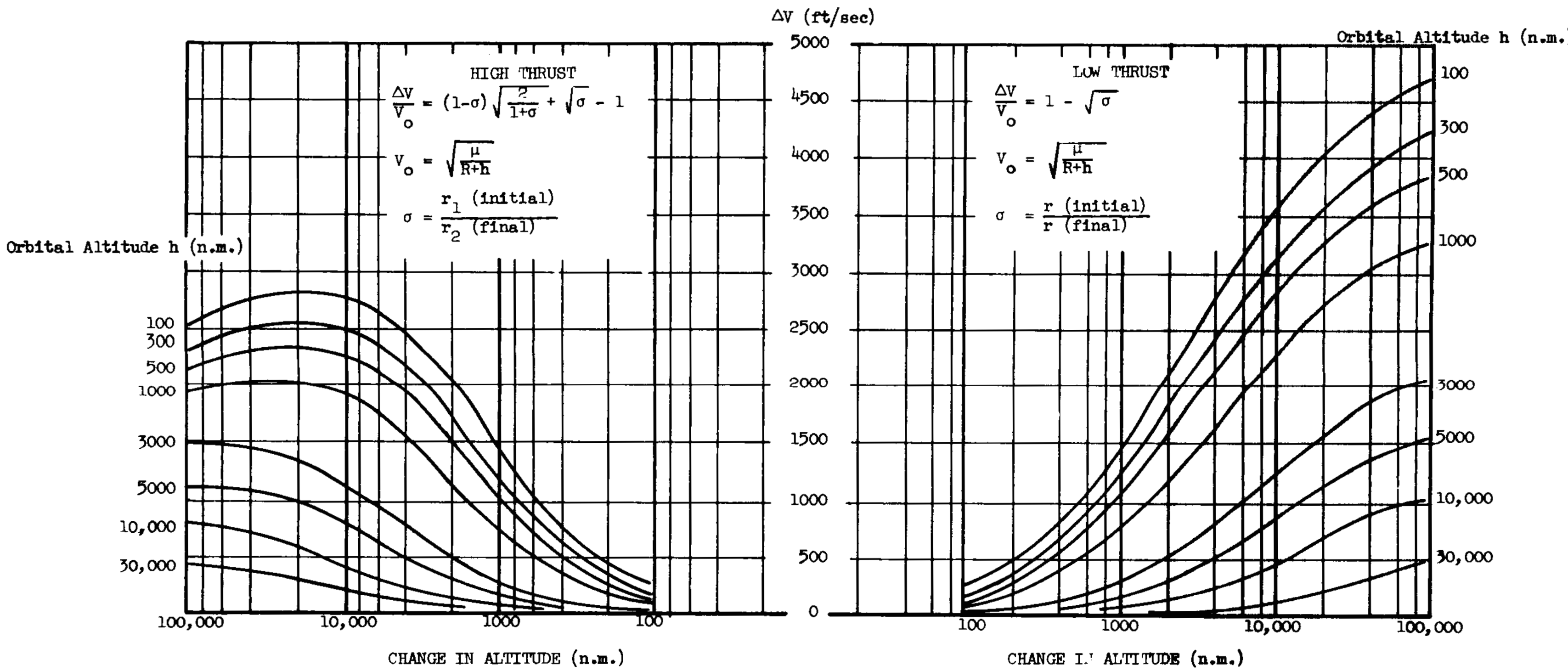


FIGURE A-25. ΔV Requirements for Altitude Change
Moon Orbits

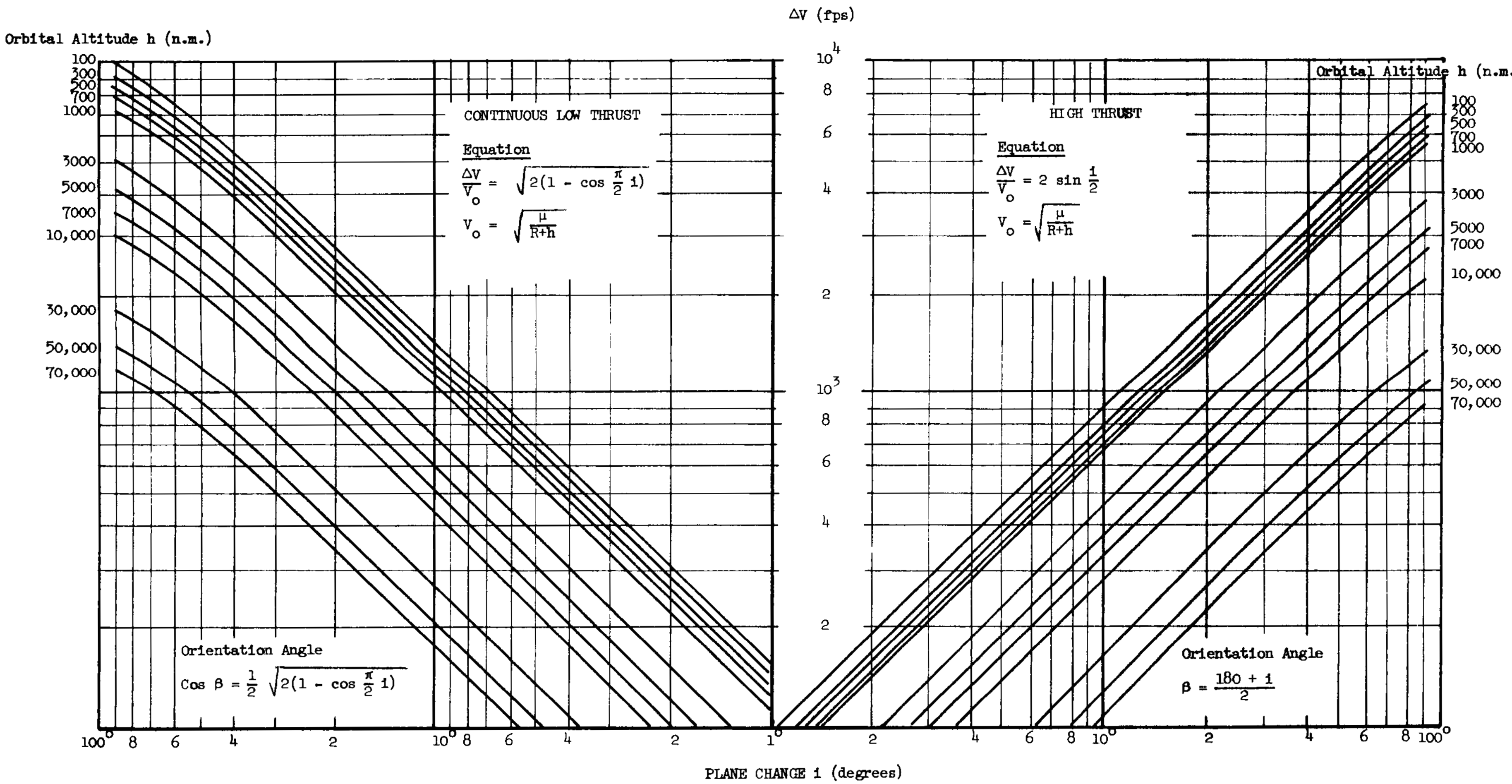


FIGURE A-26. Plane Change ΔV Requirements
Moon Orbit

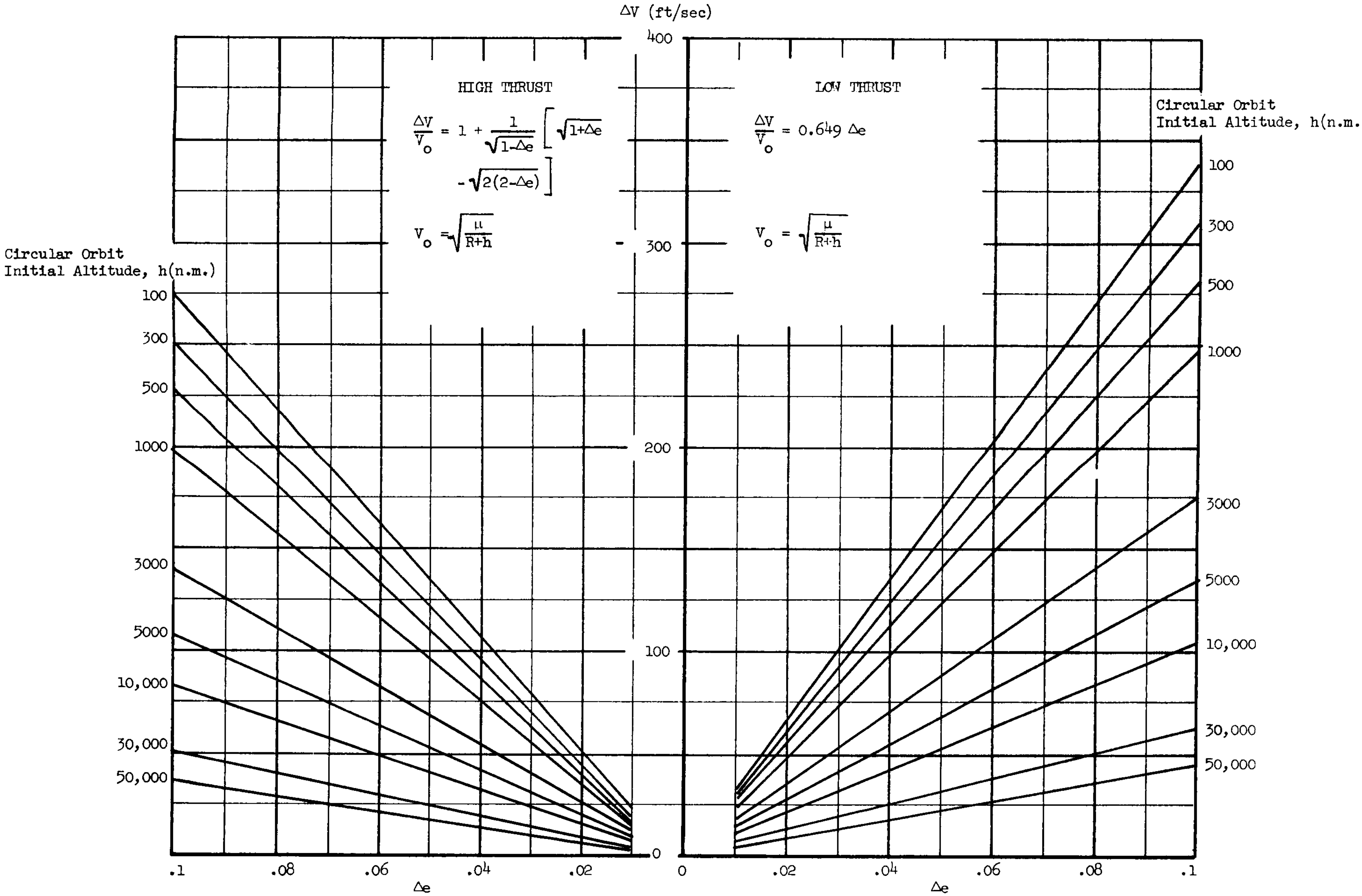


FIGURE A-27. Removal of Orbit Eccentricity
Moon Orbits

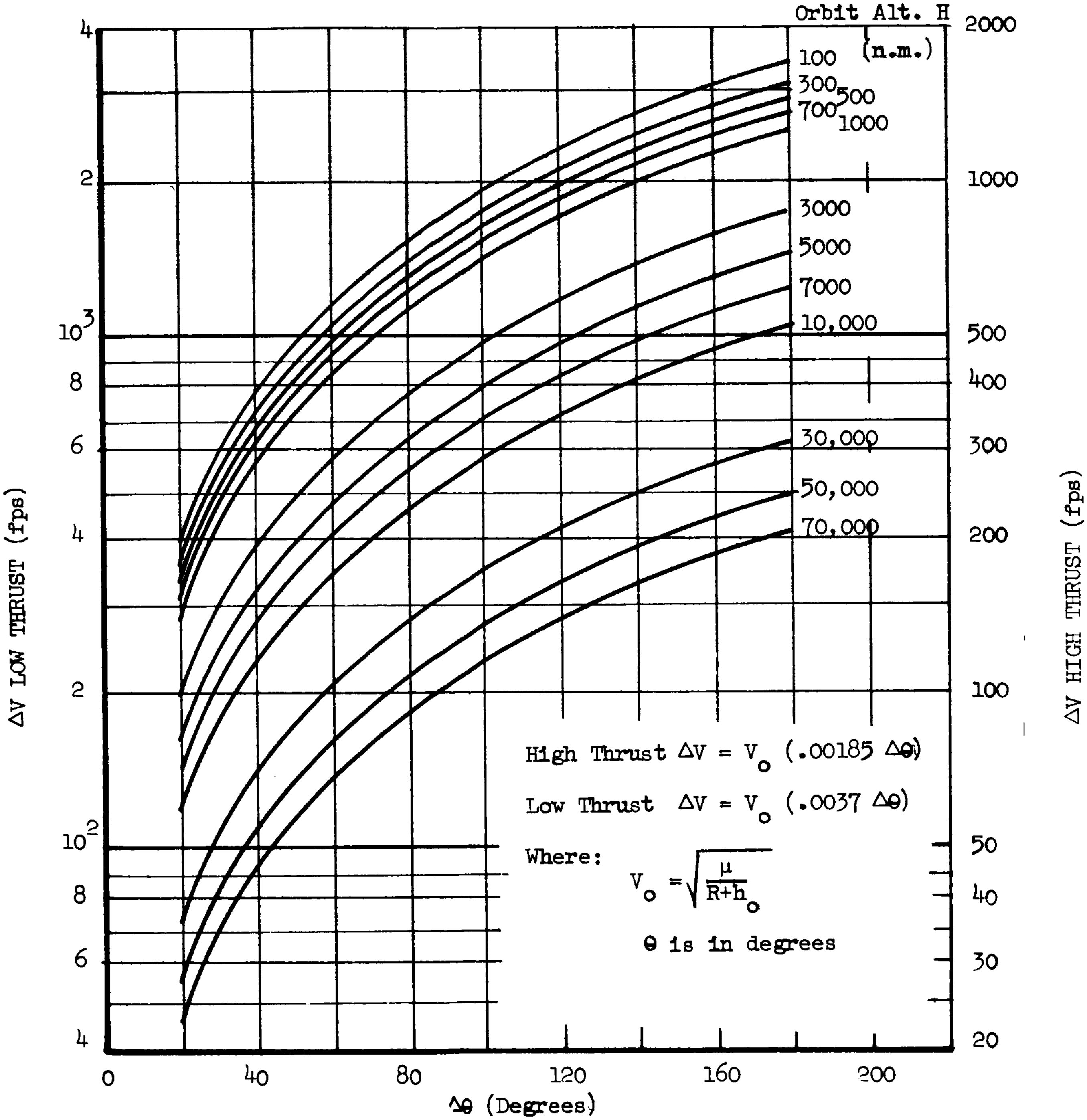


FIGURE A-28. Orbital Position Change Requirements
Moon Orbits

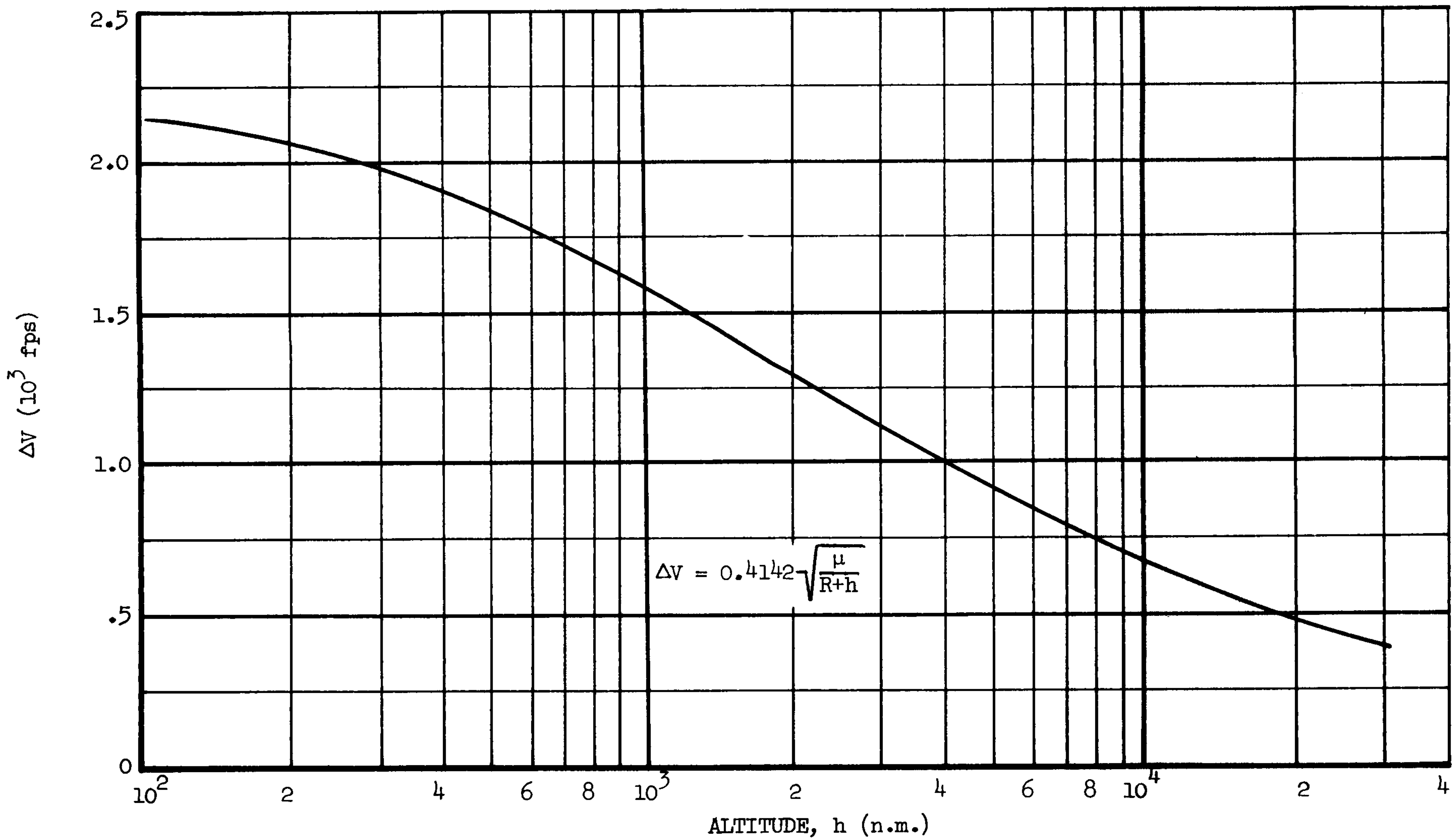


FIGURE A-29. Escape from Moon Orbit - High Thrust

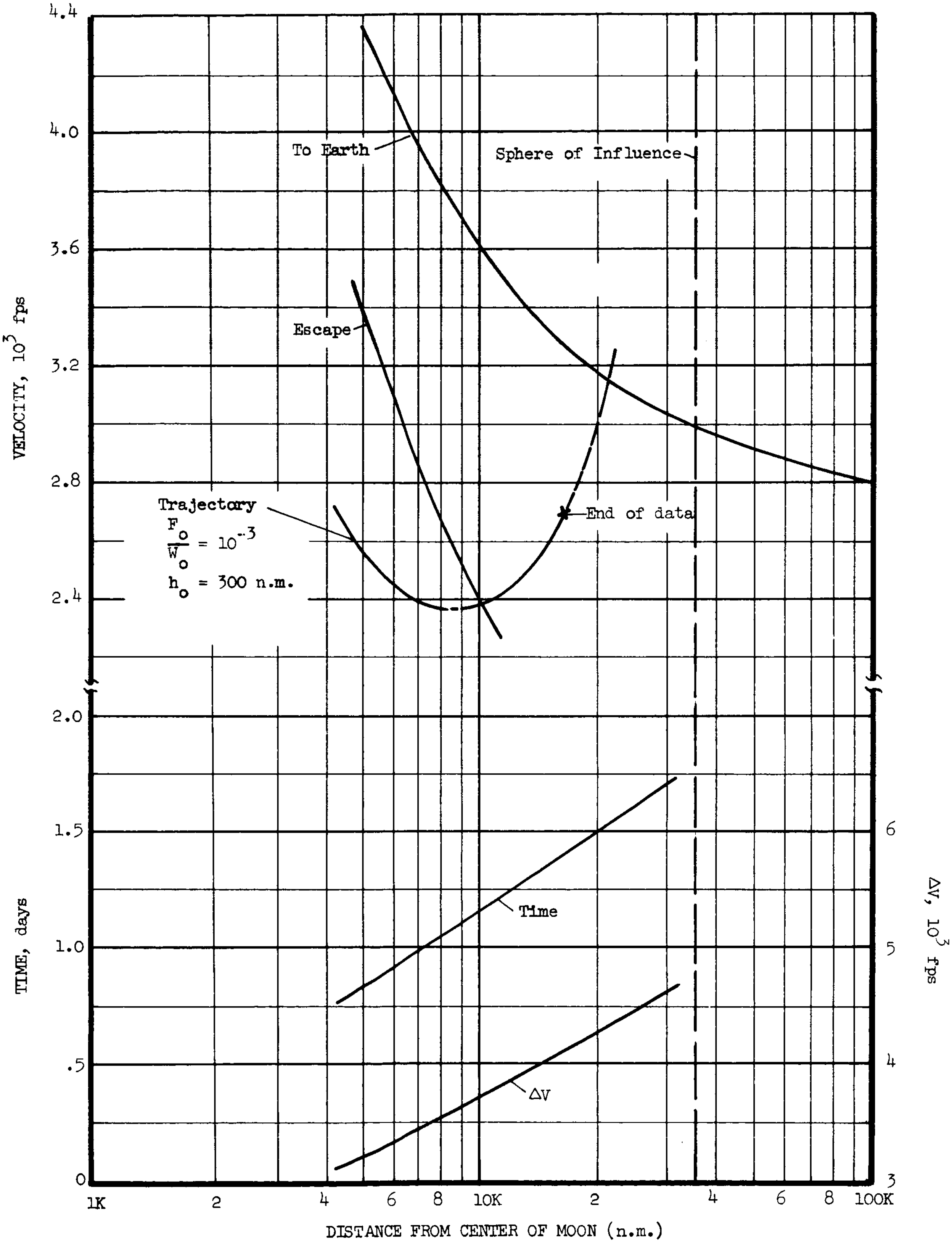


FIGURE A-30. Trajectory and Requirements for Transfer to Earth from the Moon

MISSION REQUIREMENTS

WITH MARS AS

THE CENTRAL BODY

FIGURES A-31 THROUGH A-36

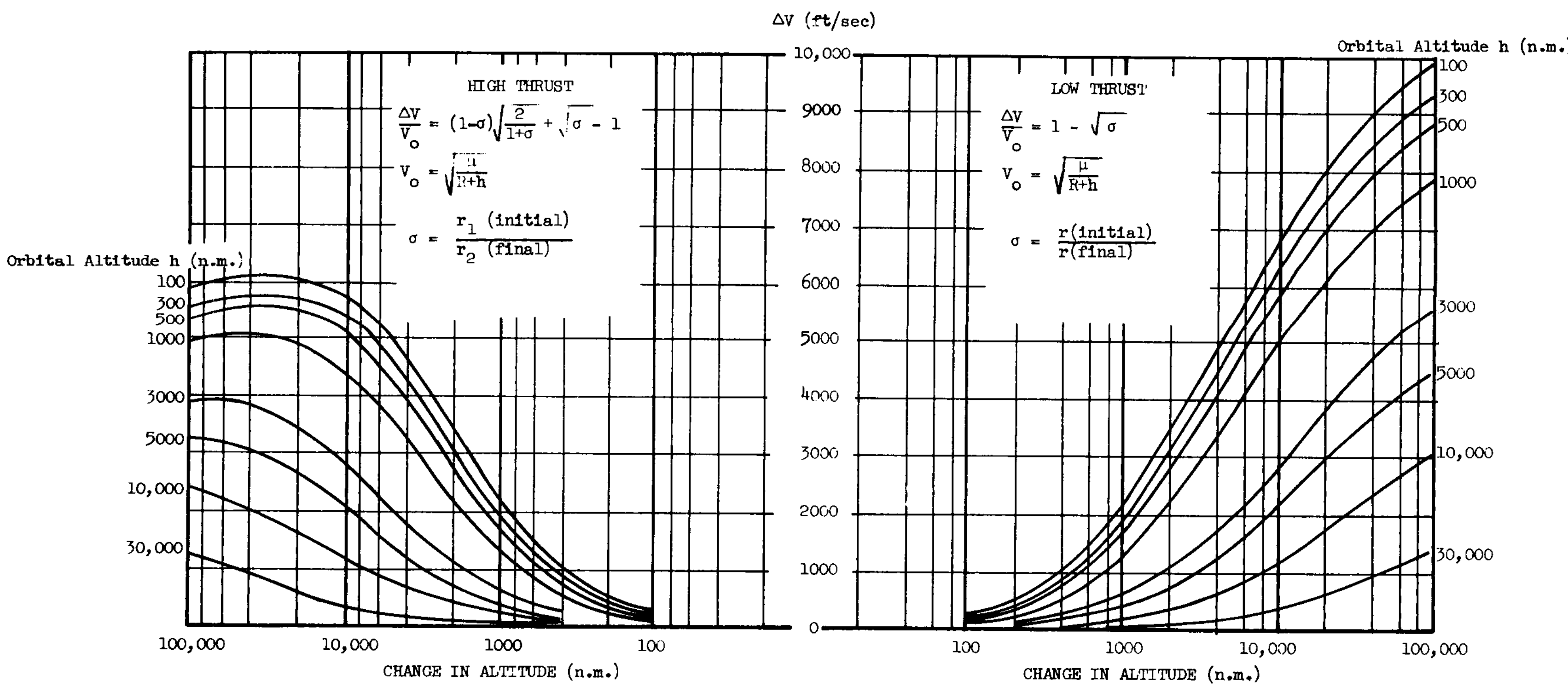


FIGURE A-31. ΔV Requirements for Altitute Change
Mars Orbits

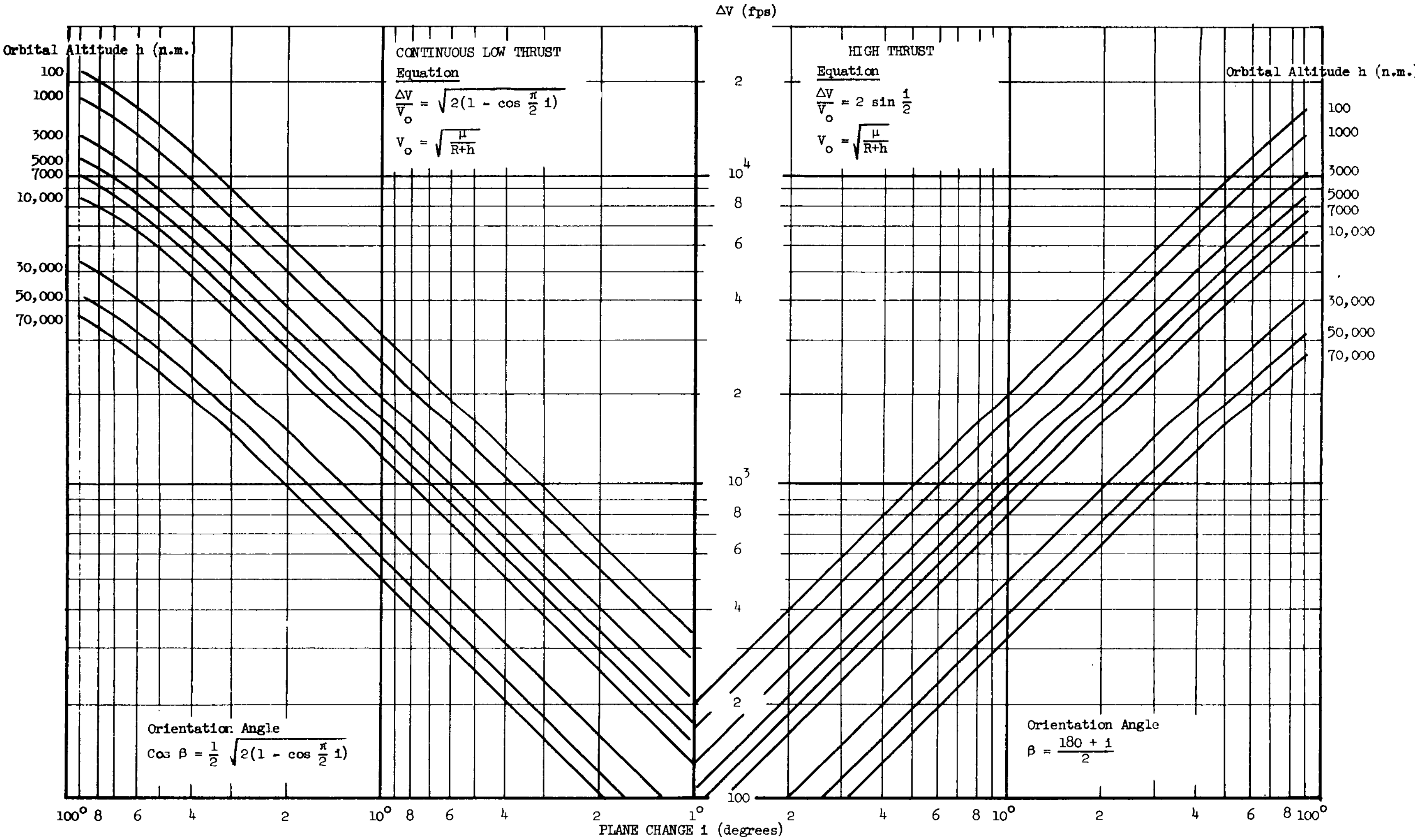


FIGURE A-32. Plane Change ΔV Requirements
Mars Orbit

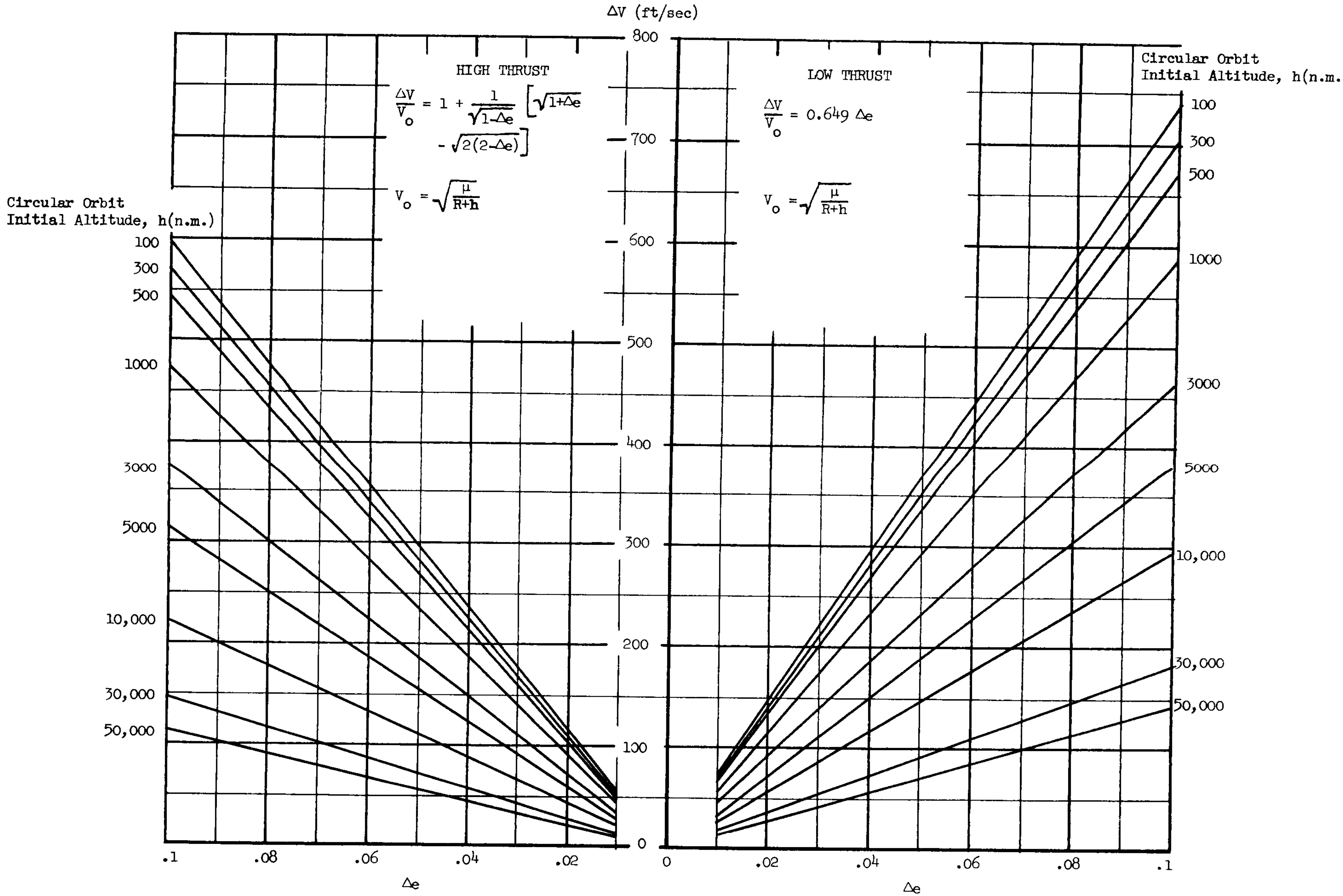


FIGURE A-33. Removal of Orbit Eccentricity
Mars Orbits

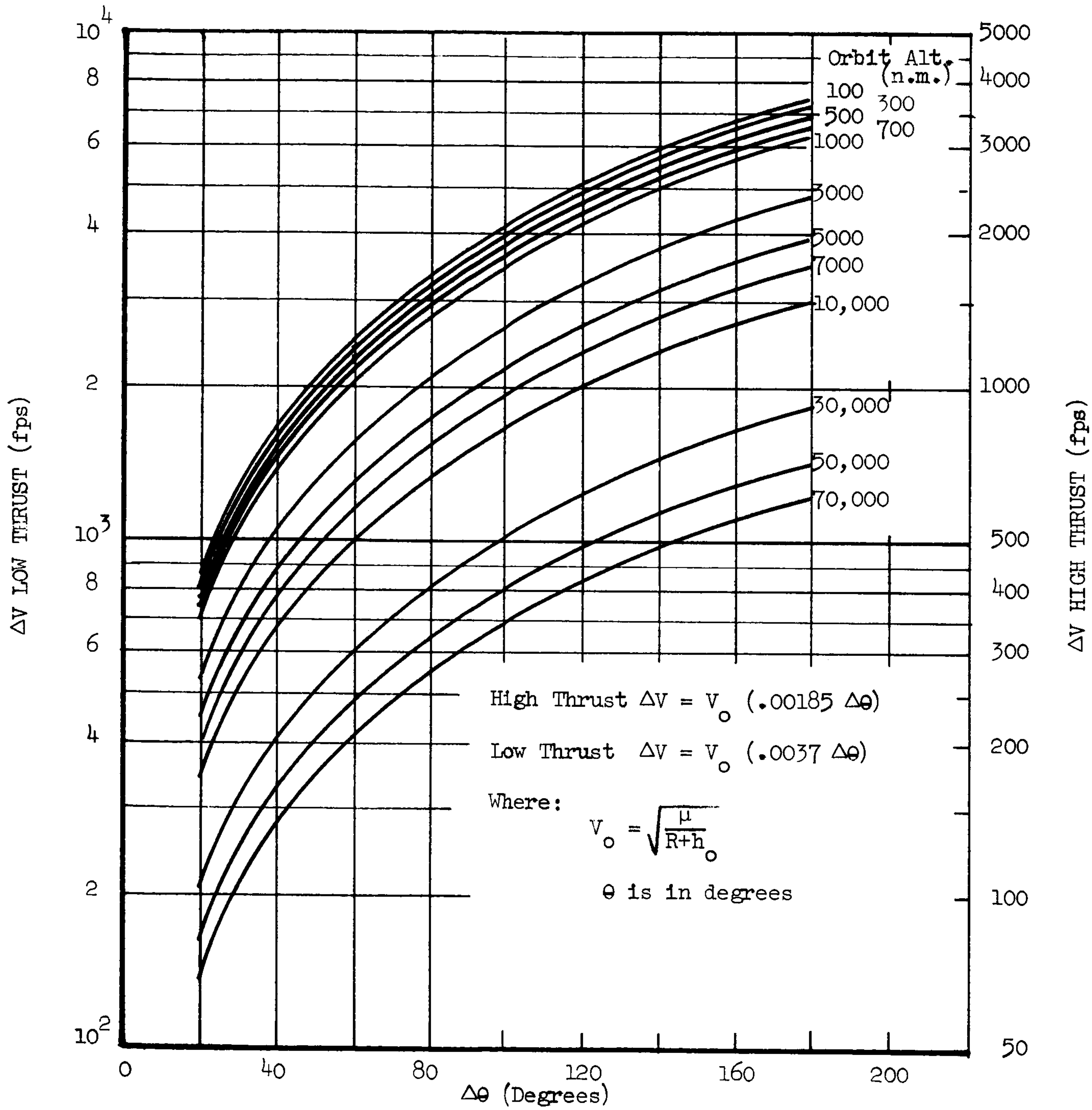


FIGURE A-34. Orbital Position Change Requirements
Mars Orbits

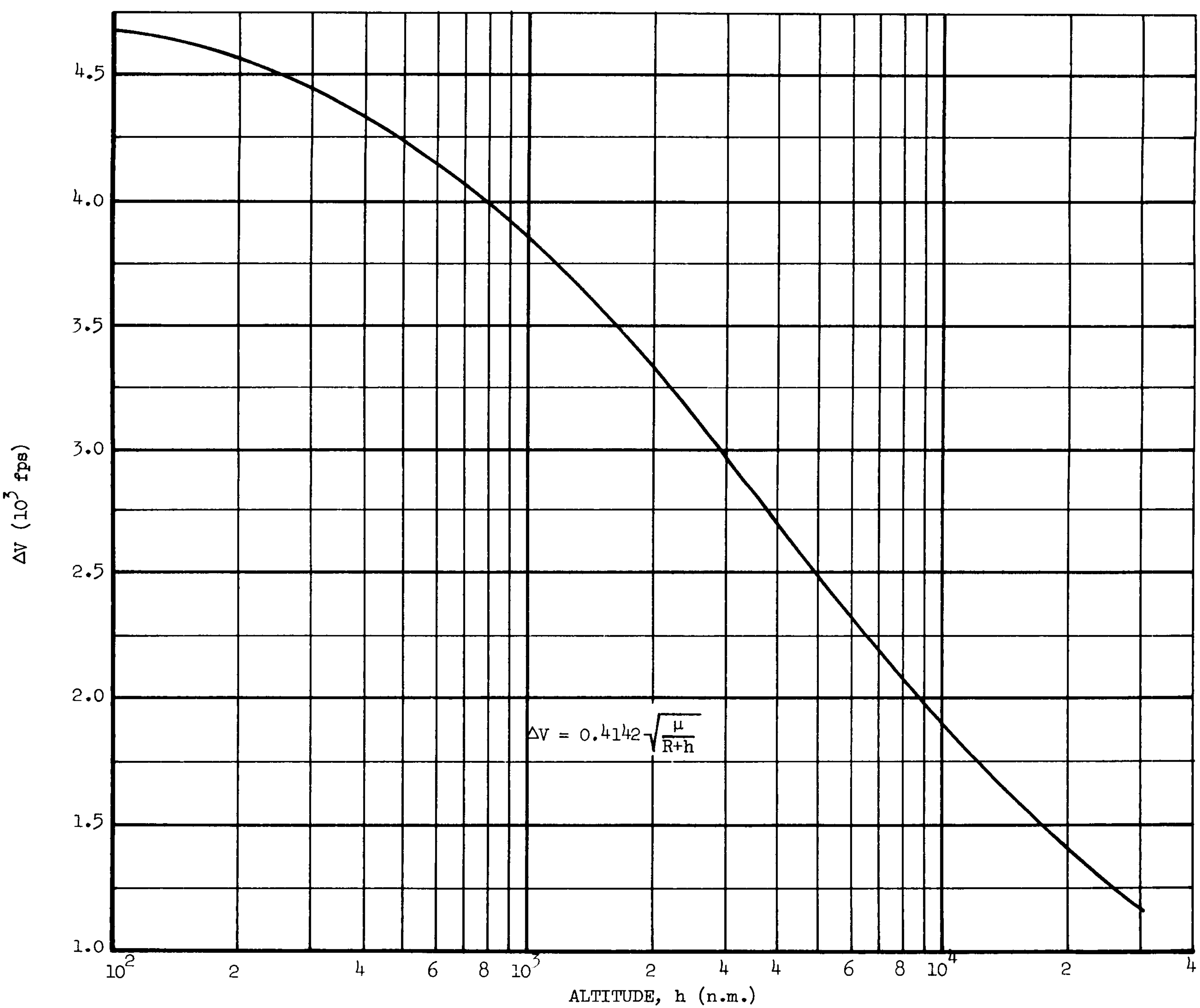


FIGURE A-35. Escape from Mars Orbit - High Thrust

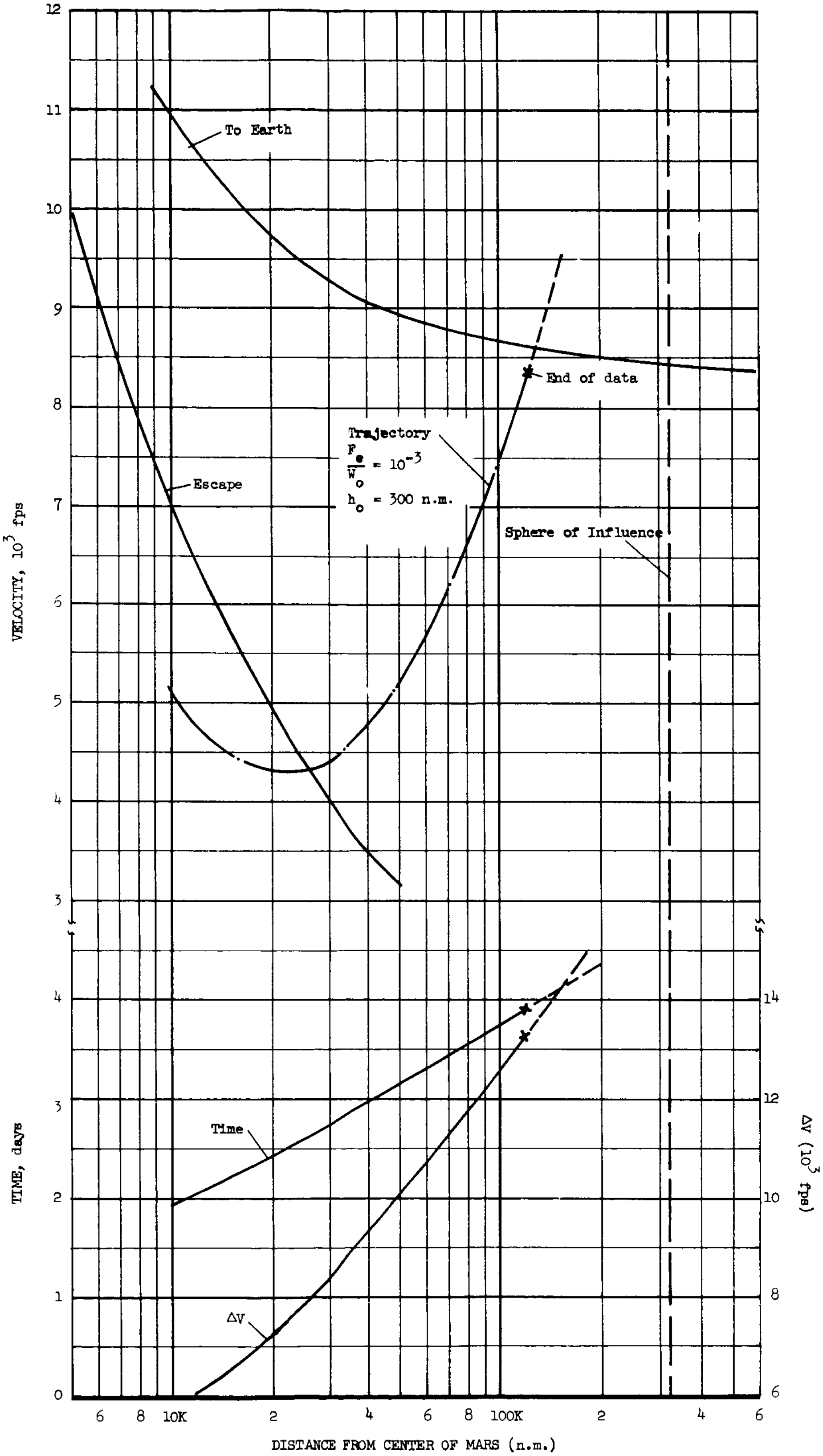


FIGURE A-36. Trajectory and Requirements for Transfer to Earth from Mars

MISSION REQUIREMENTS

WITH THE SUN AS THE

CENTRAL BODY

FIGURES A-37 THROUGH A-40

NOTES ON HELIOCENTRIC MANEUVER DATA

This section contains analytically determined ΔV and time requirements for heliocentric maneuvers employing high thrust and low thrust (characteristic of ion engines); and numerically computed ΔV and time requirements for heliocentric maneuvers employing thrust-to-weight ratios characteristic of SHPS. Figure A-37 presents the SHPS data and Figure A-38 presents ΔV data for high thrust. The transit times for high thrust are very nearly one-half of the period of the coast ellipse, which is approximately the transit time indicated by the 10^{-3} curve of Figure A-37. Figure A-39 shows the ΔV requirement for low thrust execution of heliocentric orbit transfers. Figure A-40 shows the ΔV and burning times required for changing the plane of a heliocentric orbit at one astronomical unit. For comparison, the ΔV for high-thrust performance of the same task is:

$$\frac{\Delta V}{V_o} = 2 \sin \frac{i}{2}$$

where V_o is Earth orbital velocity (97,800 fps), and i is the plane change angle.

The results represented in this section and throughout the report are based upon the planetary data of Table A-2. Table A-2 shows orbital velocity, sphere of influence, and hyperbolic excess velocities for each of the planets. Data are given with respect to the Sun for all bodies except the Moon. The data regarding the Moon are with respect to the Earth.

TABLE A-2

MISCELLANEOUS DATA REGARDING THE PLANETS

Planet	Orbital Velocity (fps)	Sphere of Influence (10^3 nm)	Hyperbolic Excess Velocity Requirements (fps) to: Mercury	Venus	Earth	Mars	Moon
Mercury	157,200	59.1	--	--	31,582	--	--
Venus	115,000	203.0	--	--	8,883	--	--
Earth	97,800	489.0	-24,741	-8,195	--	9,670	Sub-escape
Mars	78,900	314.5	--	--	-8,369	--	--
Moon	3,340	35.7	--	--	-2,721	--	--

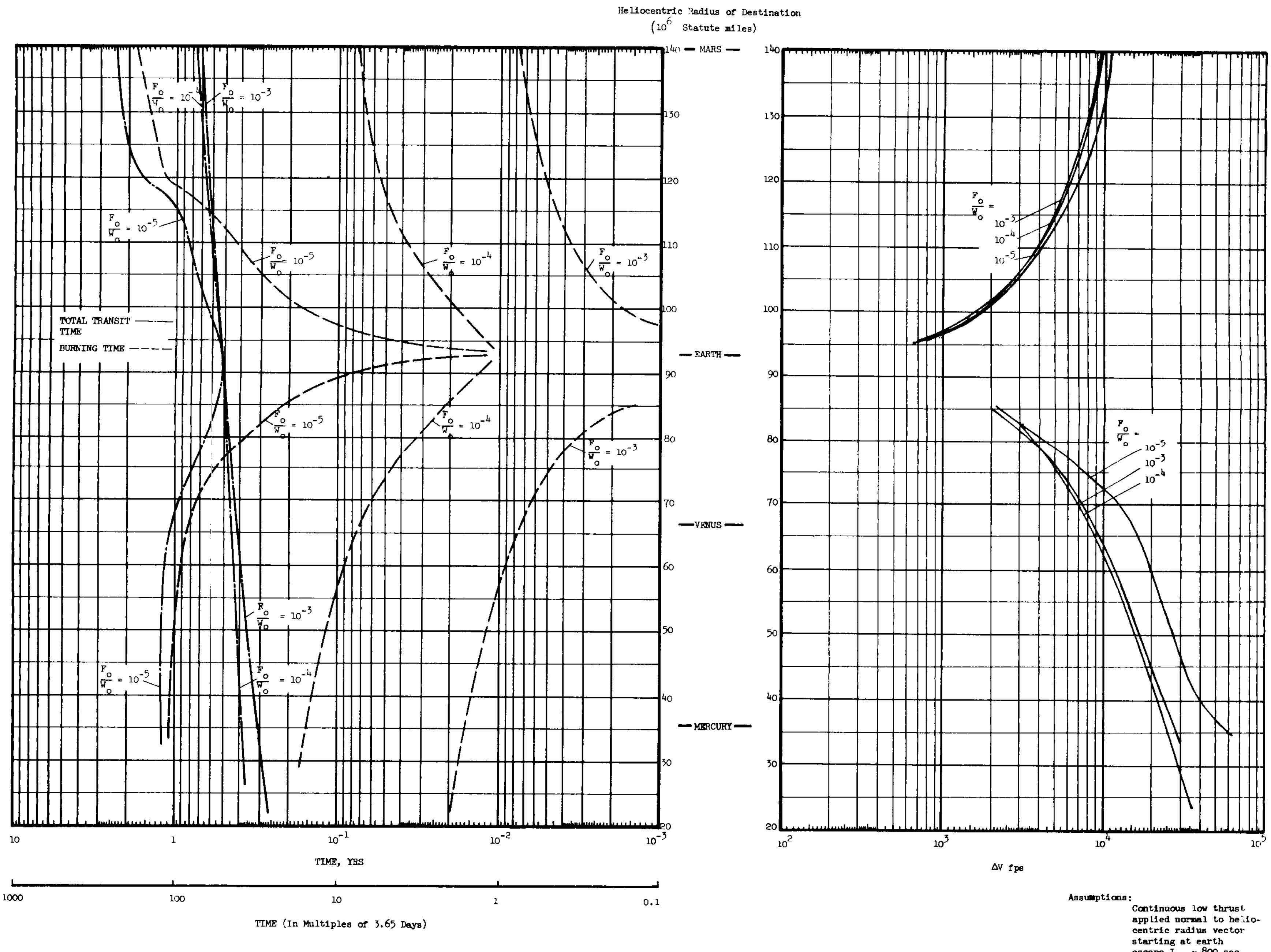


FIGURE A-37. ΔV and Time for Solar Probes

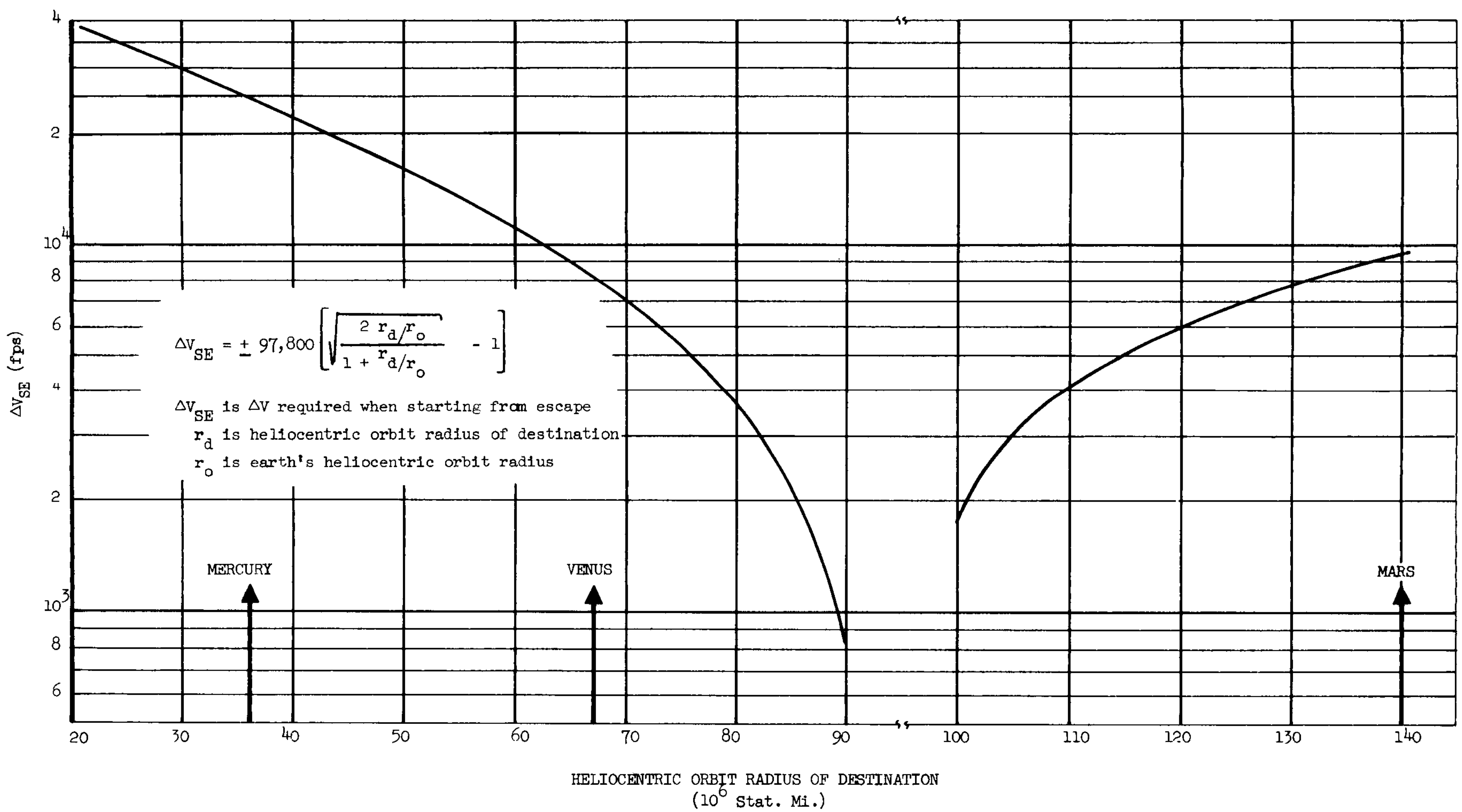


FIGURE A-38. ΔV for High Thrust Probes Starting from Earth Escape

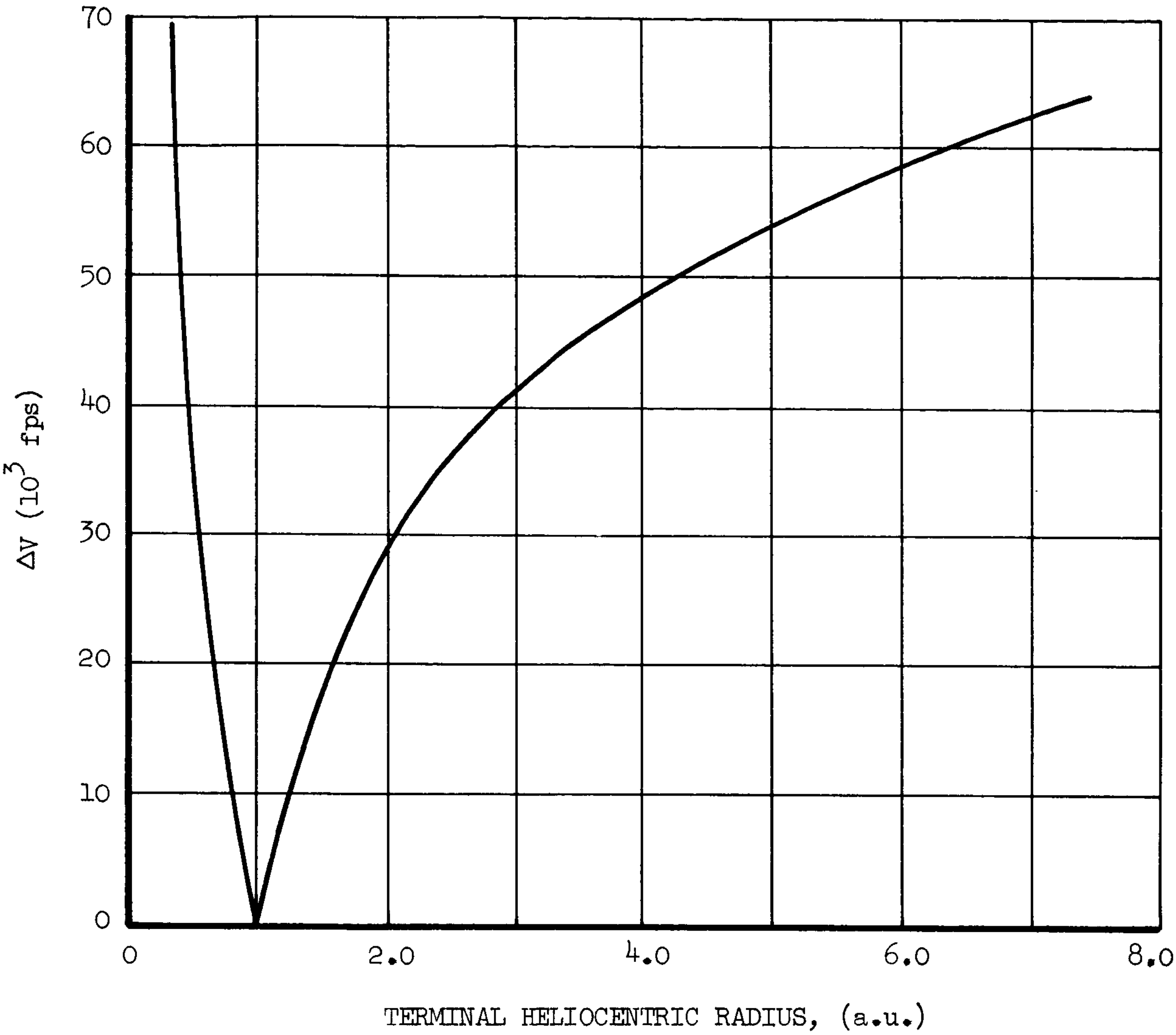


FIGURE A-39. Low Thrust Heliocentric Transfer Requirements
Earth Co-Orbit to Heliocentric Circular Orbit
(Closed form solution - negligible eccentricity)

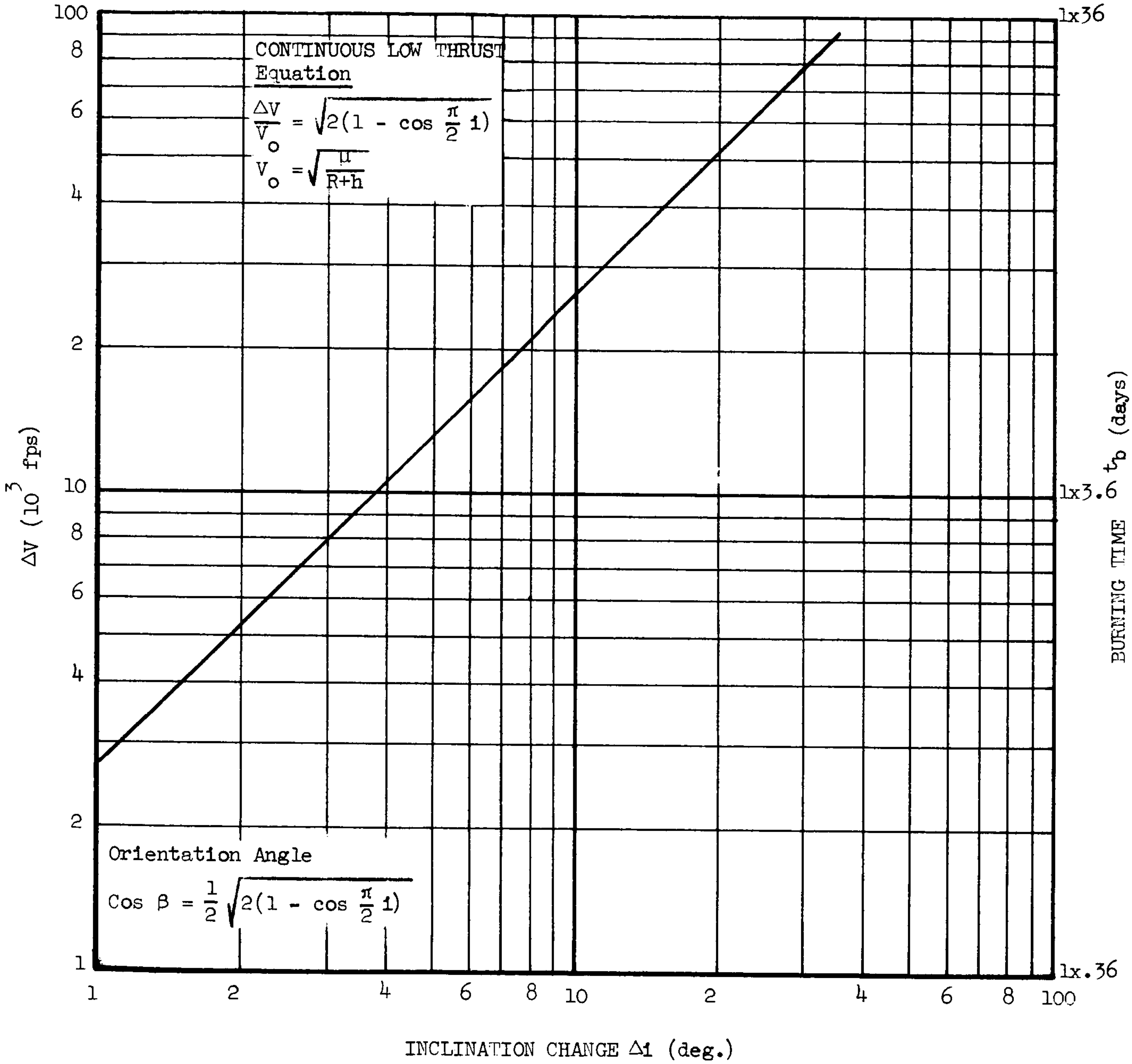


FIGURE A-40. ΔV and Time Requirements for Heliocentric Orbit Plane Change (Out-of-the-Ecliptic) at 1 a. u.

ATTITUDE CONTROL

Figure A-41 presents complete propulsion system performance requirements for attitude control. The data are most readily applied to limit-cycle (steady state) operation, but also yield requirements for counteracting estimated disturbances and performing re-orientation maneuvers. The outputs obtainable from the curves are thrust level, burning time (per cycle and total), and duty cycle, per axis. Constant-thrust attitude control and constant vehicle mass were assumed. The inputs needed to use the curves are attitude tolerance, minimum detectable angle and angular rate, vehicle weight and dimensions, and mission duration.

In the lower right hand plot of Figure A-41, the duty cycle is obtained once the attitude tolerance and minimum detectable attitude angle are known. The total burning time can then be found for any given lifetime or mission duration. The difference (θ_F) between the attitude tolerance and the minimum detectable attitude angle can be used with the attitude rate in the left-hand plot to obtain the minimum angular acceleration required and the burning time per quarter cycle (burning time per cycle is four times this number). The product of burning time and angular acceleration is "angular ΔV" which can be converted to the conventional ΔV by multiplying by the factor r_g^2/r_m. This factor is characteristic of the shape of the vehicle and independent of its weight. The thrust level requirement can be obtained for any set of vehicle weight and dimensions over a wide range by entering the upper right-hand plot with angular acceleration. The curves are identified by the product of vehicle weight, W, and a dimension factor, X. The quantity X reduces to vehicle radius r for spherical vehicles and the roll axis of cylindrical vehicles. The plot has been restricted to spherical and cylindrical vehicles.

Thus the propellant weight or total impulse requirement for the attitude control system can be obtained through application of the ΔV to

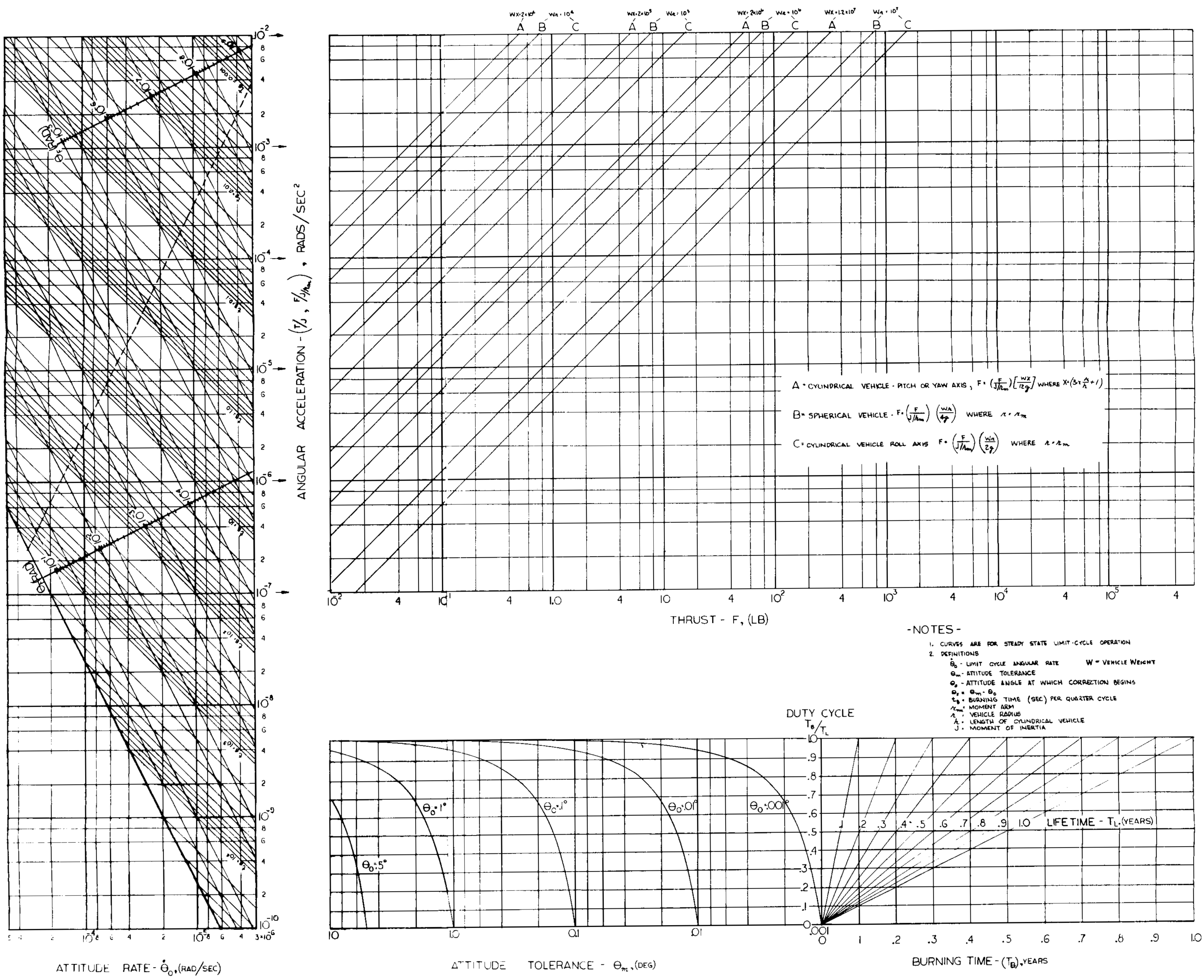


FIGURE A-41. Thrusting Requirements for Attitude Control the curve of Section I or by finding the thrust in Figure A-41 and multiplying by the total burning time. The former method will yield propellant ratio, which can then be multiplied by any selected vehicle weight to obtain propellant weight. The latter method yields only propellant weight, not propellant ratio. The pertinent attitude control equations are presented in the remainder of this section.

The equivalence between translational characteristic velocity ΔV and characteristic angular velocity $\Delta\dot{\theta}$ can easily be demonstrated:

Translational: $$F = ma \qquad (1)$$

$$Ft = m \; adt = m\Delta v \qquad (2)$$

Rotational: $$T = Fr_m = I\ddot{\theta} = mr_g^{\,2}\,\ddot{\theta} \qquad (3)$$

$$Tt = Fr_m t = mr_g^{\,2}\,\ddot{\theta}\; dt = mr_g^{\,2}\Delta\dot{\theta} \qquad (4)$$

$$Ft = \frac{mr_g^{\,2}\Delta\dot{\theta}}{r_m} \qquad (5)$$

Combining 2 and 5,

$$\frac{r_g^{\,2}}{r_m}\,\Delta\dot{\theta} = \Delta v \qquad (6)$$

Now, it can be shown that

$$\Delta\theta = \frac{T_L\dot{\theta}_o^{\,2}}{2\theta_m - \theta_o} \qquad (7)$$

Substituting in 6,

$$\Delta v = \frac{r_g^{\,2}\,T_L\,\dot{\theta}_o^{\,2}}{r_m(2\theta_m - \theta_o)} \qquad (8)$$

This ΔV can be inserted in standard propulsion equations to determine propellant and payload ratios.

The ratio of attitude control thrust to vehicle mass is

$$\frac{F}{m} = \frac{\dot{\theta}_o^{\,2}}{2(\theta_m - \theta_o)} \; r_g^{\,2}/r_m \tag{9}$$

Equations 8 and 9 require only the shape information of the vehicle, not the size or mass. If vehicle mass is known, the ΔV concept is not required. Including vehicle mass in the input data results in:

$$J = m\, r_g^{\,2} \tag{10}$$

$$\frac{r_g^{\,2}}{r_m} = \frac{J/r_m}{m} \tag{11}$$

$$\Delta V = \frac{J/r_m \; T_L \dot{\theta}_o^{\,2}}{m\,(2\theta_m - \theta_o)} \tag{12}$$

The total burning time is

$$T_b = \frac{2(\theta_m - \theta_o)\; T_L}{2\theta_m - \theta_o} \tag{13}$$

and the thrust requirement is

$$F = \frac{\dot{\theta}_o^{\,2}\; J/r_m}{2(\theta_m - \theta_o)} \tag{14}$$

from which the total impulse and propellant weight can be obtained directly.

List of symbols:

r_g	=	radius of gyration
r	=	moment arm
T_L	=	lifetime requirement of system
θ_m	=	attitude tolerance (half angle)
θ_o	=	attitude sensor threshold
$\dot{\theta}_o$	=	limit cycle angular rate
m	=	vehicle mass

J = moment of inertia

T_b = total burning time

Requirements for offsetting disturbance and for re-orientation of a space vehicle are determined as follows. The maximum anticipated disturbance force and its duration must be estimated, and the maximum allowable angular displacement due to that force must be selected. The disturbance force can be considered as a thrust and the resulting values of angular displacement θ_F and angular velocity $\dot{\theta}$ found in the left-hand plot, with the known duration identified as t_b. The thrust required depends upon the delay allowed before actuation. The displacement allowed should be such that the steady-state thrust level can be employed, but for a longer duration than that required for normal limit cycling. The impulses required for counteracting disturbances is generally slightly greater than the integrated impulse of the disturbances. In addition to reducing the resulting angular velocity to zero, energy is needed for neutralizing the integrated attitude error. This energy requirement is calculated in the same manner as the requirement for vehicle re-orientation; it is directly proportional to the angular velocity with which the attitude discrepancy must be removed and independent of the attitude error already accumulated. The requirements for each phase of attitude correction (limit cycling, removal of rates, and introduction or removal of displacements) can be found in Figure A-41 and totalled to obtain the over-all vehicle requirement. Each displacement is considered θ_F, each rate $\dot{\theta}_o$, each force F, and the duration of each force, t_B.

STATION KEEPING

The requirements for station keeping of a vehicle in a circular orbit are presented in Figure A-42. The data are presented in terms of ΔV or propellant weight per year plotted against orbital altitude, with frontal area as a parameter in the case of drag-type perturbations. Figure A-43 presents an approximate correlation between frontal area and vehicle weight for a number of representative shapes and sizes.

The station keeping requirements are based on the assumption that the perturbing forces must be exactly neutralized over the operational lifetime of the satellite. Corrections for drag perturbations are in terms of total impulse which is the time integral of the drag force. The requirements for gravity perturbations are in terms of ΔV, which is the time integral of the perturbing acceleration, such accelerations being the same for all satellites regardless of weight or dimensions. The total impulse and ΔV requirements can both be easily converted to propellant weight, when analyzing a mission.

The thrust-level requirements involved in station keeping are extremely small. The actual forces and accelerations are so low that it would be impractical to attempt to counteract them continuously. Rather, a periodic correction would be made at a relatively high thrust level (1 or 2 pounds). The correction would be made frequently enough to keep the vehicle within its specified operating zone and the correction time would be short compared with the time of uncorrected orbital motion.

The frequency and duty-cycle of the station keeping system depend on requirements of the individual satellite mission, and are much less significant than ΔV and total impulse for purposes of sizing vehicles and payloads.

However, it is important to note that the station keeping system will be called upon to restart many time, and to remain operable after long periods of time in space. The operational profile of the station keeping system can be presented in parametric form with such quantities as orbital position accuracy and sensing accuracy as parameters, in a manner similar to that of attitude control. However, it was not possible to do this under the current contract funding limitations.

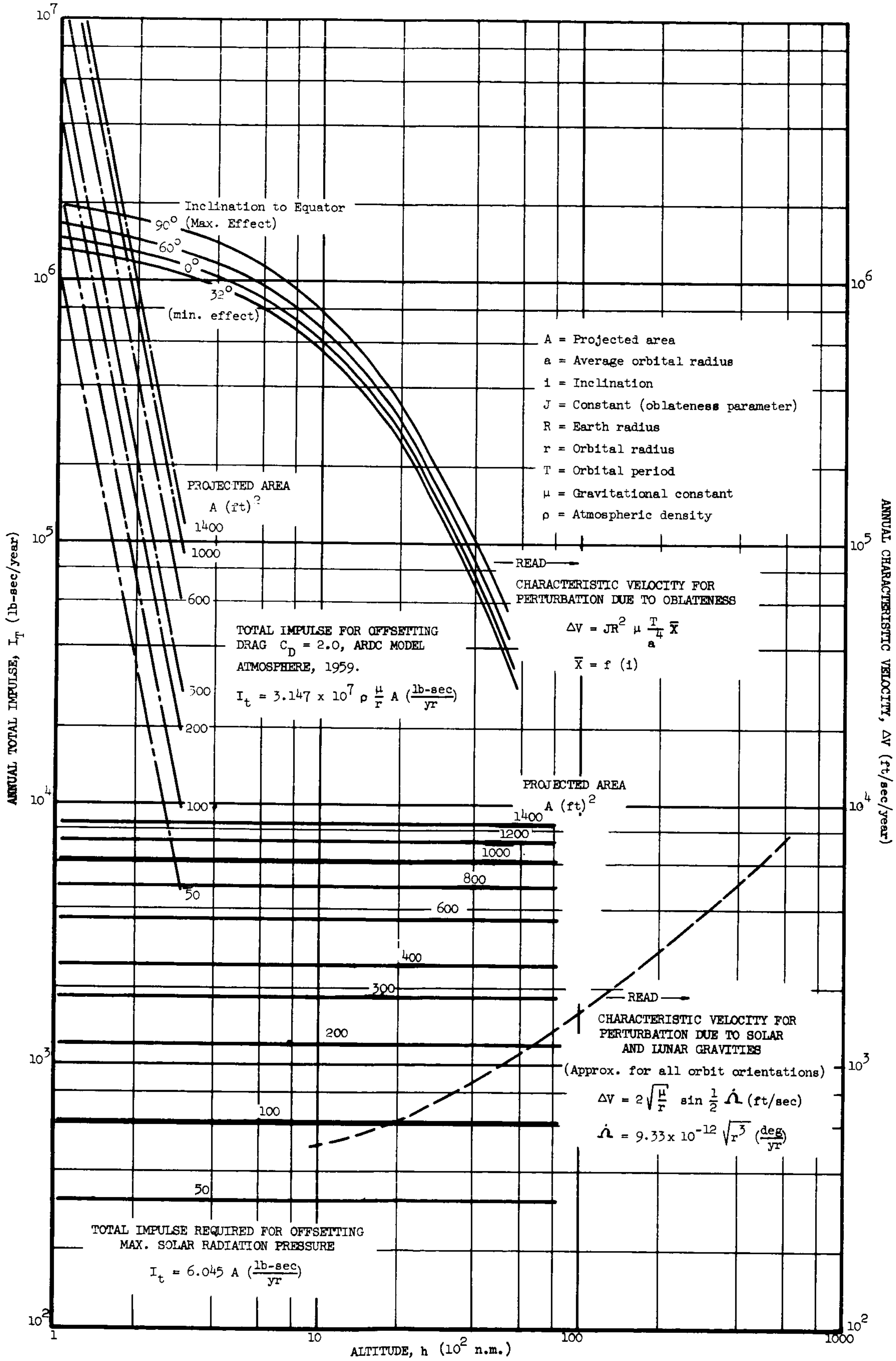


FIGURE A-42. Station-Keeping Requirements for Satellites

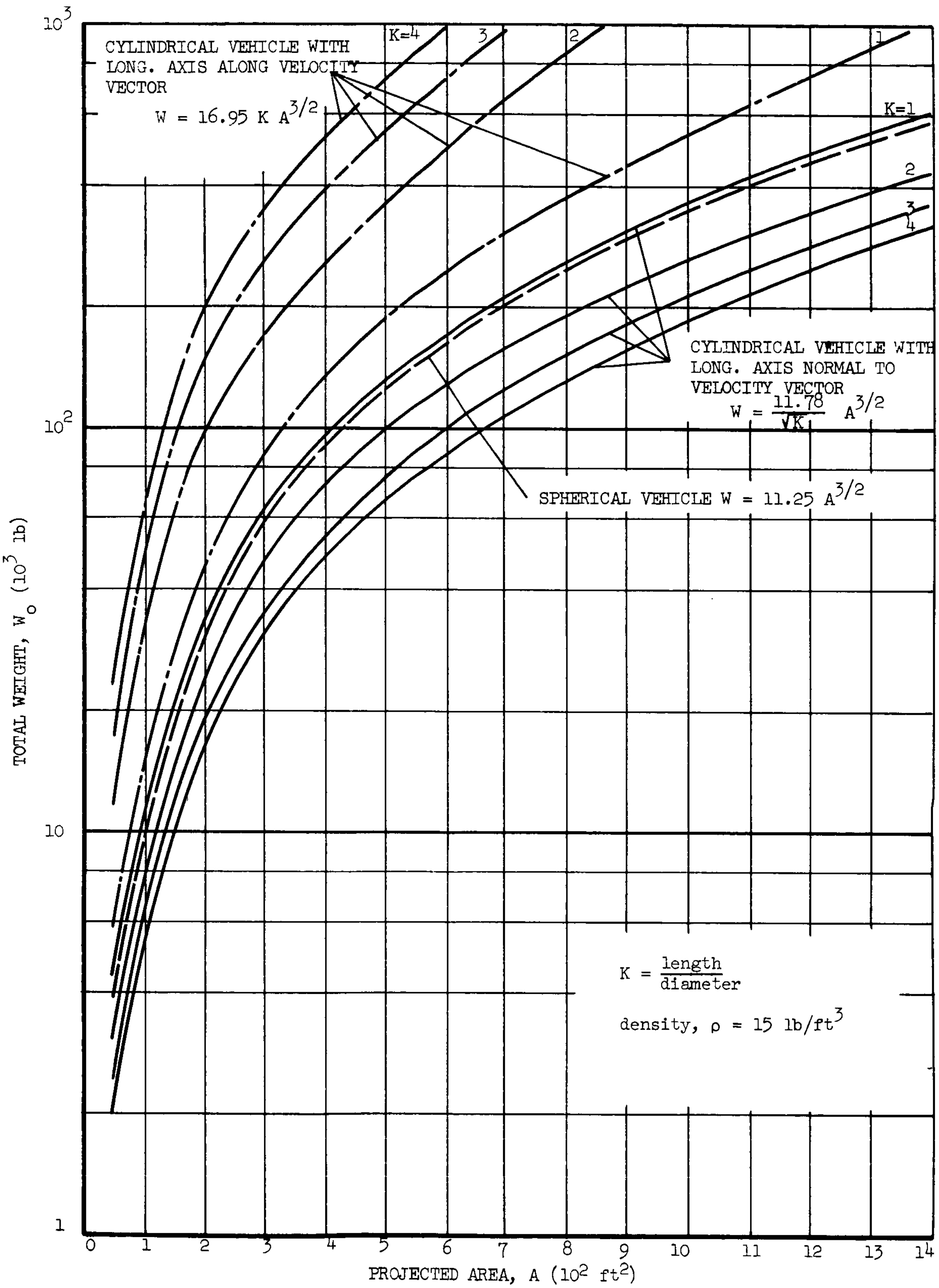


FIGURE A-43. Vehicle Total Weight vs Projected Area for Drag Calculations

SHADOW TIME

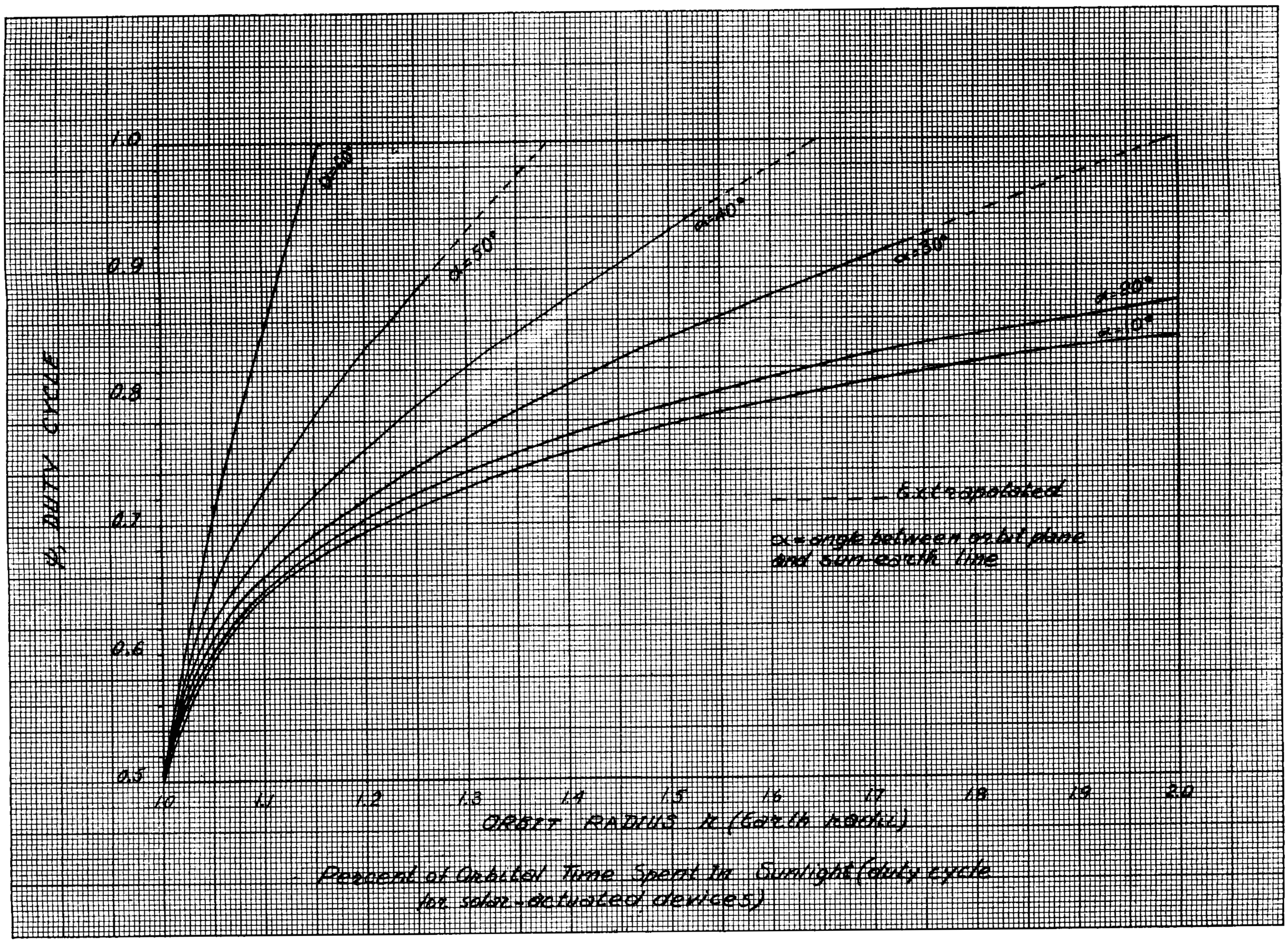
An obvious limitation of any solar-powered device is that it cannot operate efficiently if the Sun's rays are blocked out by another object. The storage of the thermal energy for use during periods of darkness has not proved to be a simple matter. It is important, then, to consider carefully the effect of this shadow time on the fulfillment of a mission by a SHPS.

It is evident that the effect of shadow time on heliocentric phases of interplanetary missions is negligible and the effect on planetocentric phases is small at the high altitudes. In fact the shadow time for earth satellites is never greater than 10 percent for orbits above 7740 nm altitude, regardless of inclination (Figure A-44).

The presence of the shadow affects the mission analysis in two principal ways. It increases the elapsed time required to perform a maneuver, over that which would be required if truly continuous thrusting were possible. In addition, it restricts the process of thrusting to certain parts of the orbit. This restriction can be quite significant, such as in the case of an inclination change to be performed on an orbit whose node is in the shadow (the most effective thrusting for changing inclination takes place at the node). Another case in which the shadow effect may be significant is in the execution of an eccentricity correction. For such a maneuver the most effective thrusting takes place in the neighborhood of the apsides.

For the operation of transferring from one orbit altitude to another, the shadow effect is manifested primarily as a cause of increased mission time. The increase in transit time resulting from the shadow effect is indicated in Figure A-45. For the case shown, transfer from a 100 nm earth orbit to any orbital altitude, the transit time is increased by as much as 20 percent due to the shadow effect. The curves were obtained by integrating by hand the average shadow time for successive bands of altitude. Where the curves become parallel, the effect of shadow time has essentially disappeared. This occurs, for orbits inclined 40° (angle α) or more to the Sun-Earth line, at approximately 2,000 nm altitude. It does not occur for orbits of zero angle (those whose planes contain the Sun-Earth line) until an altitude of approximately 20,000 nm is reached.

Unfortunately, the angle between the orbit plane and the Sun-Earth line does not remain constant for a given orbit as the Earth revolves about the Sun. To account for the change in this angle during the orbit transfer maneuver, assume that the angle changes in steps at certain intervals of time. Using the applicable curves for the appropriate time intervals the total transfer time can be obtained. The time interval covered in switching from one curve to another at a fixed altitude should not be included in the total time for the transfer operation. An example, including the effect of varying angle is shown by the heavy line in Figure A-45, the transfer between curves being indicated by a dashed line. The total transfer time is the sum of the time intervals spanned by the heavy lines. The total time is, of course, always less than the time corresponding to the smallest angle, occurring during the mission, and greater than the time corresponding to the largest angle occurring during the mission.



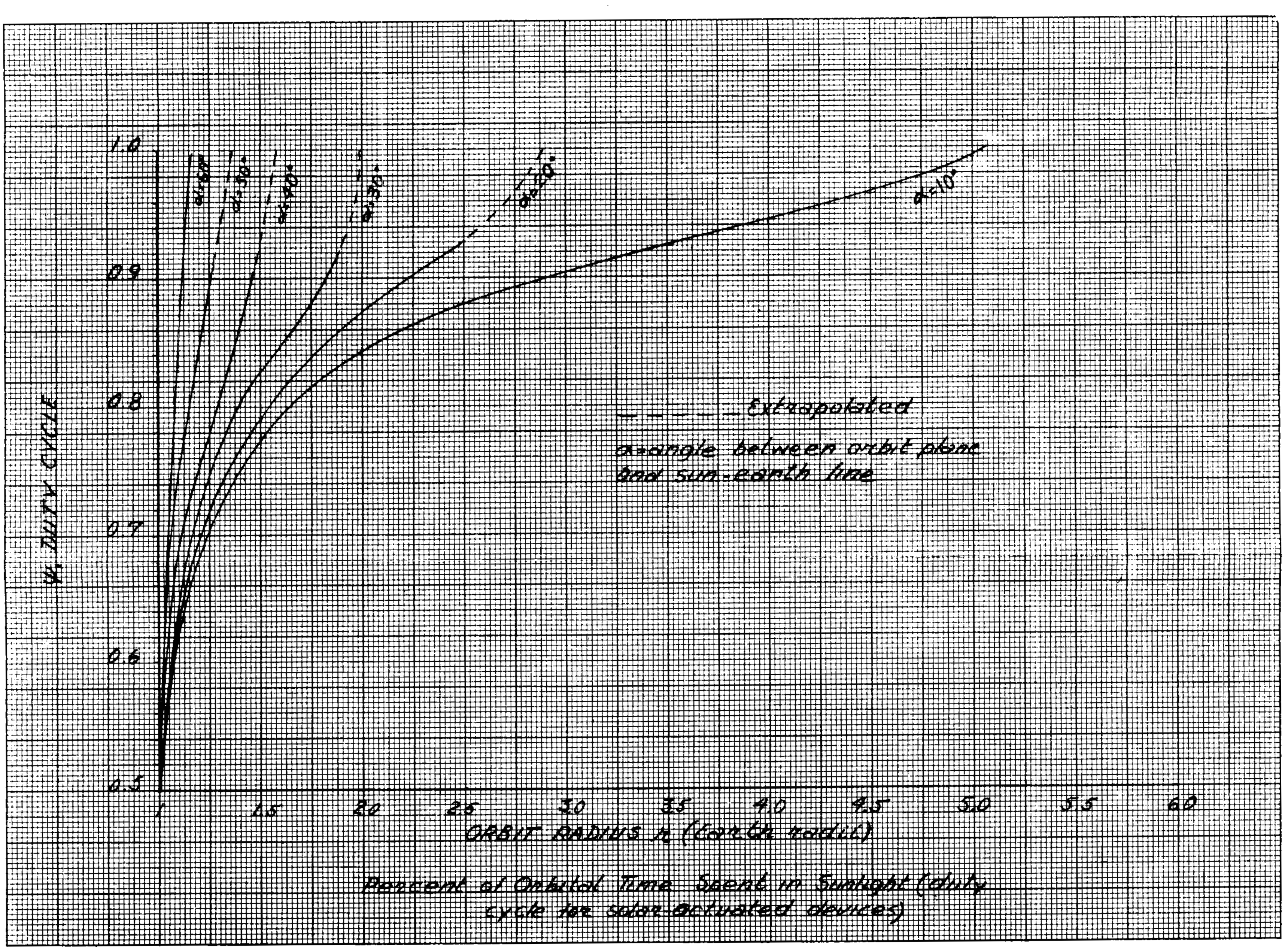


FIGURE A-44. Percent of Orbital Time Spent in Sunlight

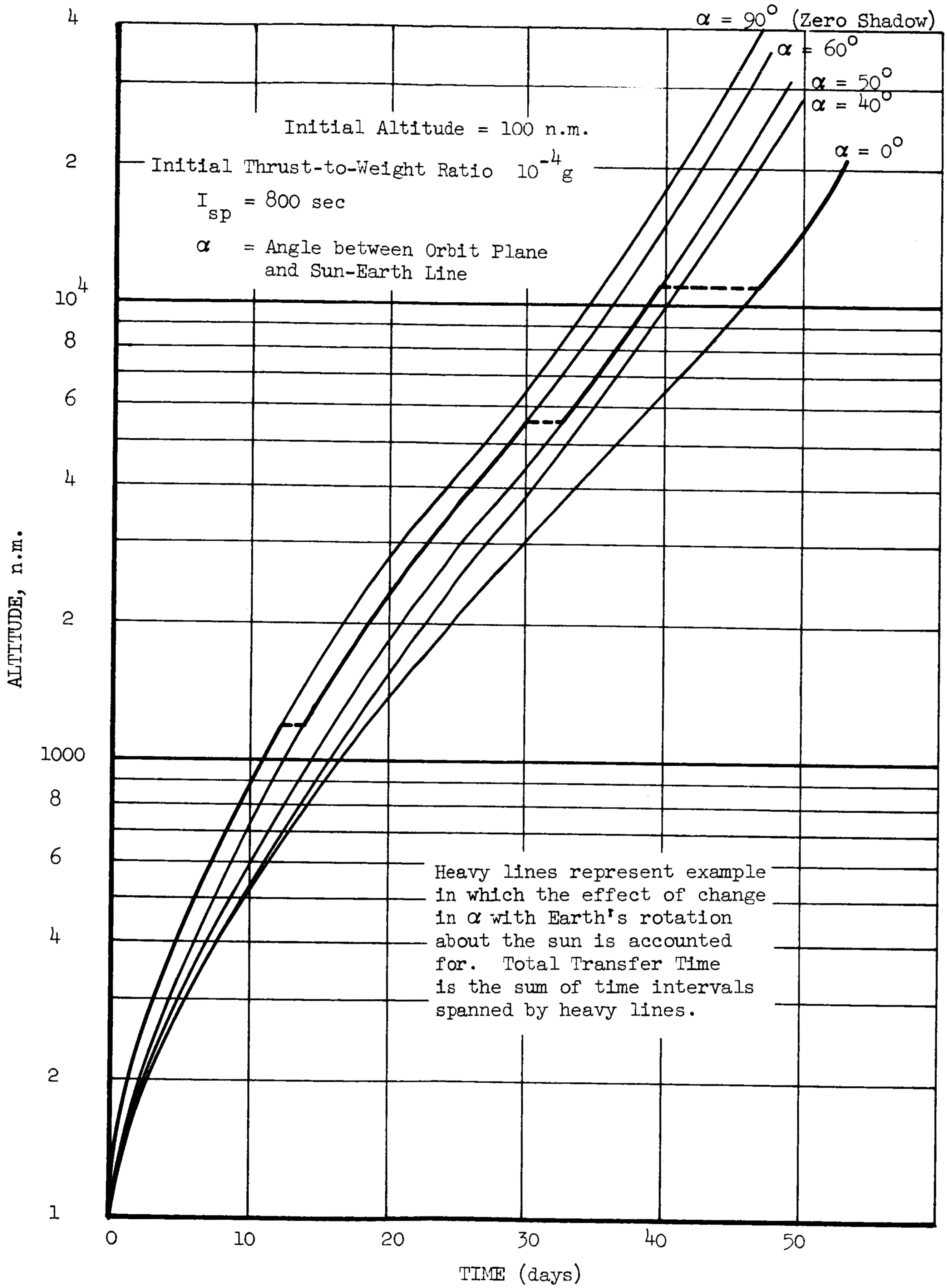


FIGURE A-45. Altitude vs Time for Low Thrust Orbit Transfer Including Effect of Shutdown in Earth's Shadow

III. SYSTEM WEIGHT AND PERFORMANCE CHARACTERISTICS

The payload capability of a propulsion system is intimately related to the weight of the inert parts of that system. In a parametric study, it is necessary to determine quickly the weight of inert parts for any given propulsion task. The inert weight can be expressed, approximately, in two simple terms: a propellant dependent term and a thrust-dependent term. The mission analysis structure developed in this study is specifically suited to accept data in such terms.

The currently assumed weight functions for the three propulsion system types of principal interest, SHPS, Chemical (LOX-H_2) and Nuclear-Hydrogen are listed in Table A-3. The function for the nuclear thrusting system weight contains an added constant which indicates the existence of a minimum weight level. The values for nuclear systems and SHPS are, at best, educated guesses at this point, since a prototype of neither has been completely designed. The estimated component weights of one SHPS configuration are shown in Table A-4. The weight of the hydrogen tanks for the SHPS, and probably the nuclear system, is relatively firm because the tank design task is relatively conventional. The weight of the thrusting system, for both SHPS and Nuclear-Hydrogen systems, is, however, highly variable. The bounds on SHPS thrusting system weight (including concentrator, absorber, nozzle, etc.) are estimated to be 87-175 $\frac{\text{lb}}{\text{lb thrust}}$ considering the various configurations. Figure A-46 is a set of propellant fraction curves based upon the weight dependencies listed in Table A-3. In establishing the payload delivered with a specific SHPS configuration an adjustment in thrusting system weight must be made. An addition of 40 lb/lb can be made when the most current estimate is considered.

Included in Figure A-46 (the lower right hand graph) is a chart which provides, for quick reference, the propulsion system weight for combinations of propellant fraction and propellant weight.

Rough order of magnitude values for nuclear-electric (ion) systems were obtained, but payloads were not calculated for those systems. Some representative values of the weight dependencies of nuclear-electric systems are

$$W_T = 0.0291\ W_P^{\frac{1}{2}}$$

$$W_F = 6200\ F$$

which apply to a system of one pound thrust. The thrusting system weight (W_F) is composed of the engine weight (200 F) and the reactor weight (6000 F). The weight dependency of the nuclear-electric thrusting system is obviously not a direct function of thrust but follows an exponential curve presumably indicative of improvement in development.

Another type of propulsion system that has been considered is the solar powered electric system. Such a system would employ an ion engine powered by an electric power supply with solar energy as the source of power. For such a system, the tankage weight and engine weight dependencies would be the same as assumed for nuclear-electric systems.

The concentrator diameter associated with systems now under development are about 42 feet, and produce approximately 15 kw. This available electric system would weigh approximately 986 lbs., of which 974 lbs. would be power supply weight and 12 lbs. would be ion engine weight. Thus, the weight of the thrusting system (W_F) would be 9740 F, which is somewhat higher than the nuclear-electric systems.

TABLE A-3

SUMMARY OF WEIGHT DEPENDENCIES USED IN THE MISSION ANALYSIS

	Tankage Weight (lb)	Thrusting System Weight (lb)
SHPS	$0.044W_p^{1.082}$	$127F_o$
Chemical ($LOX-H_2$)	$0.028W_p$	$0.053F^{.84}$
Nuclear	$0.23W_p^{0.971}$	$0.0485F_o \pm 8375$

TABLE A-4

CURRENT ESTIMATE OF COMPONENT WEIGHTS
FOR OPTICAL TUBE CONFIGURATION OF SHPS

Thrust (lb)	1	10	25
Nozzle	2.7	27	67.5
Absorber	10.4	62.7	204.5
Concentrator	67.25	672.0	1681.2
Optical Tube	7.9	79.5	198.7
Total Weight (lb)	88.3	841.7	2151.9
Total Sp. Weight (lb/lb thrust)	88.3	84.1	86.6
Concentration Ratio	16250.0	16250.0	16250.0

PROPELLANT FRACTION AND WEIGHTS FOR ADVANCED PROPULSION SYSTEMS

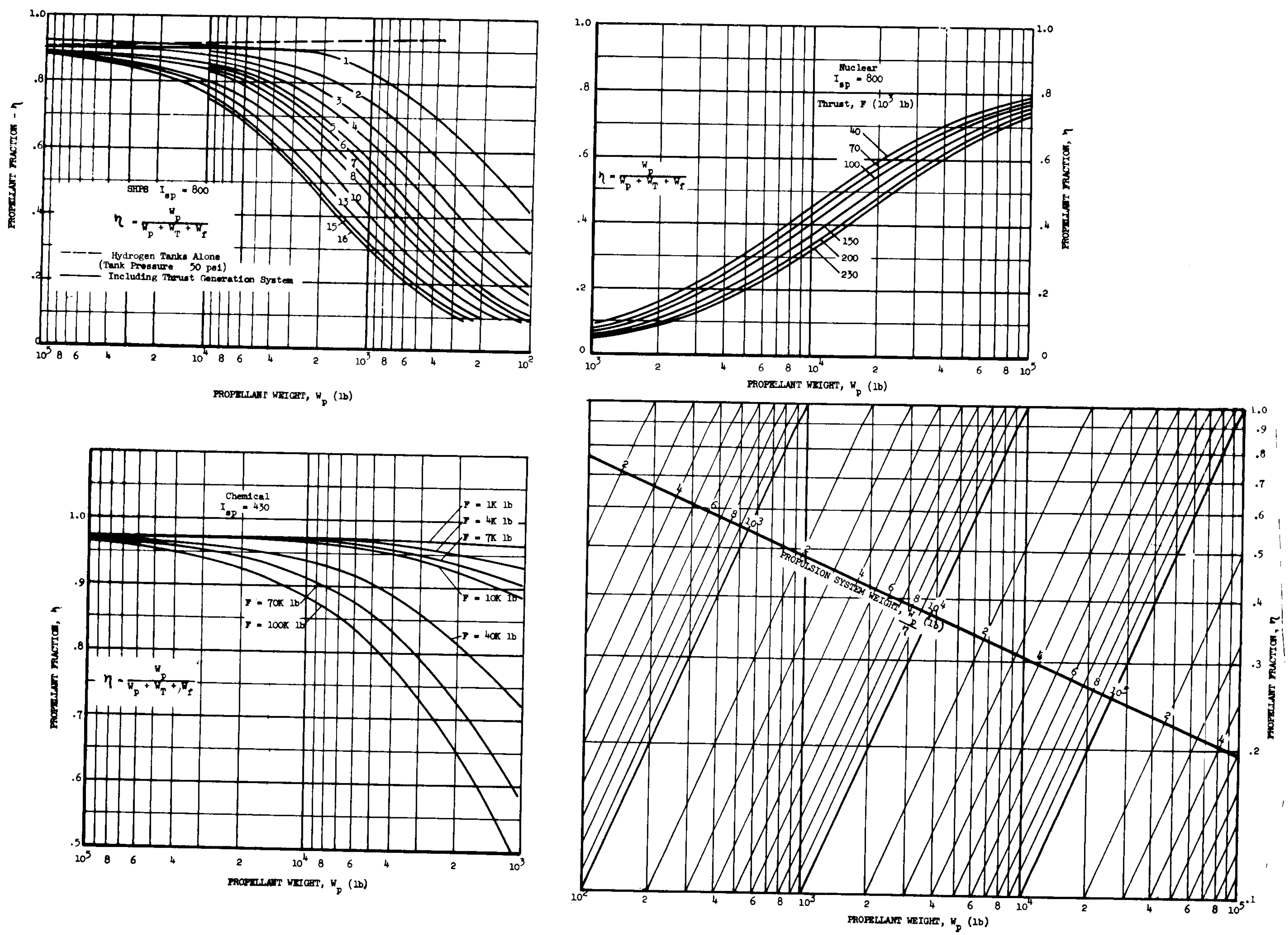


FIGURE A-46. Propellant Fraction (η) vs Propellant Weight (W_p) For Advanced Propulsion Systems